Unorthodox (heretic?) approaches for (electro)magnetic fields for compact objects

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Driving goal

- Understand both gravitational and electromagnetic wave emissions from key systems
  - Binary black holes interacting with surrounding media
  - Binary neutron stars
  - Black hole – neutron star binaries

- Connect with efforts to understand emissions from single black hole systems
  - E.g. single black hole + disk
  - AGNs, GRBs, etc.

Peculiarities

- Deal with spacetime curvature
  - Einstein equations

- Deal with fields describing fluids and electromagnetic fields
  - perfect fluid for matter
  - Electromagnetic fields? Not a unique prescription
    - Far away, EM fields in vacuum
    - Regions with $B^2 >> P$ → 'force-free' regime
    - Regions with $P >> B^2$ → inertia of matter dominates, ideal MHD regime

Work towards all 3 regimes in one ongoing (Palenzuela's talk), in the mean time, want to extract valuable information for physics and longer term goals.

Peculiarities contd

- GR related
  - Broadly speaking nothing is conserved (beyond ``constraints'')
    - Efforts to truly conserve quantities key in Newtonian (or fixed background) settings while might help are neithre required nor necessarily well defined...
    - Speed of light time step constraint.
    - For possible wide applications (physics, regions, new ideas) would like flexible methods not constraining algorithmic options [e.g. convergence and consistency only requirements]
      - Ex: DivB=0 and E.B=0 constraints → treat both at equal footings. For instance, through ``Lagrange multipliers''

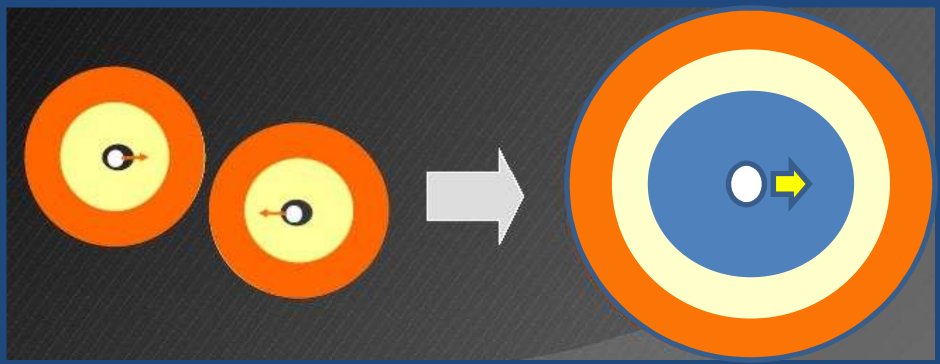
Outline. (Piece-meal approach to EM effects)

- What can curvature do?
  - Binary black holes as field stirrers
- What can curvature, and inertia of matter do?
  - Binary neutron stars
- Curvature, inertia of matter and horizon
  - Black hole – neutron stars

In all cases: Where the wild things are (i.e. fall apart)

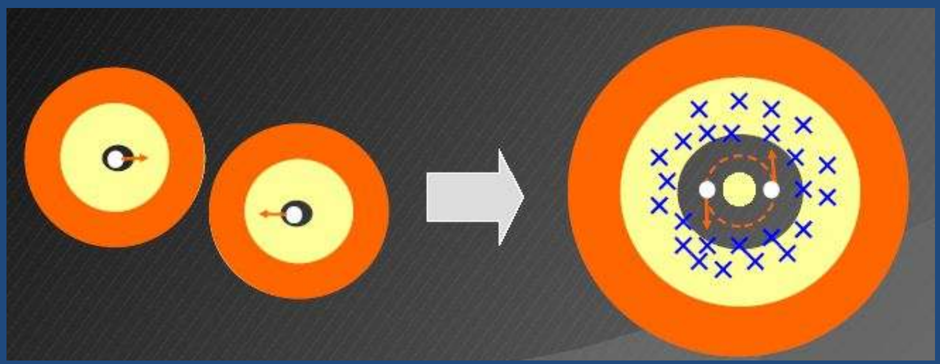
Binary black holes and emissions

- Different possible options.
  - Postmerger events from circumbinary disks around BHs



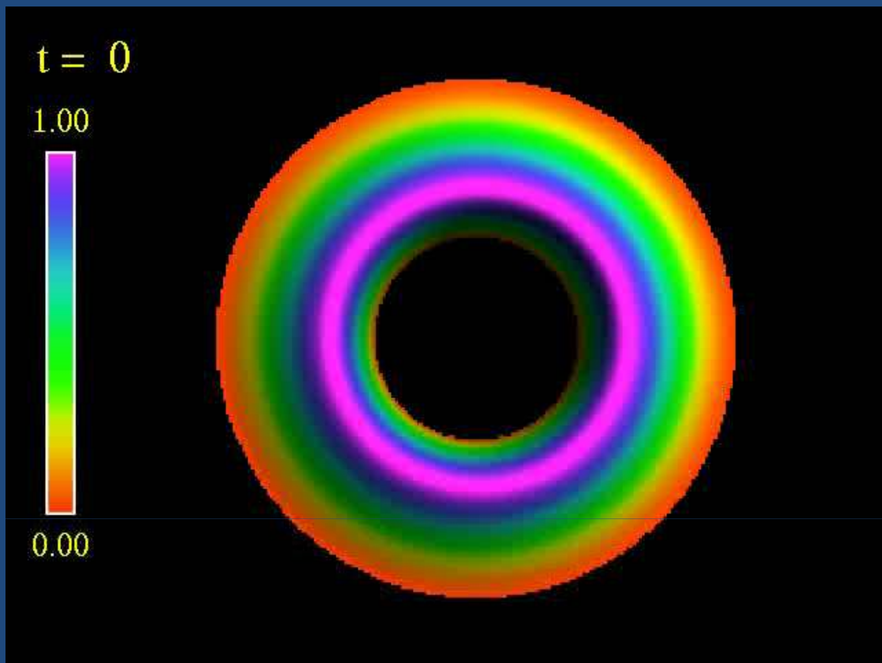
[Milosavljevic-Phinney;
Lipai-Loeb;
Lipai et.al,
Bonning et.al;
Bode et.al;
Megevand et.al]

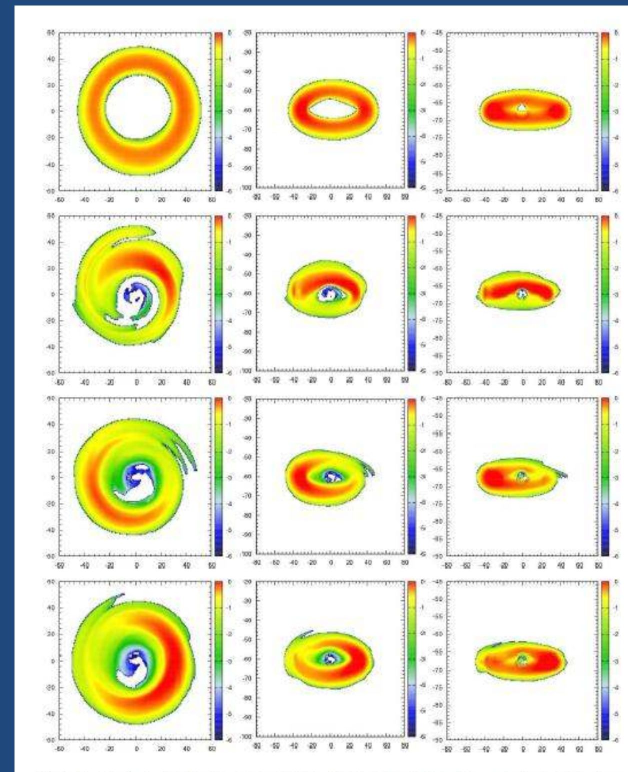
  - Pre/merger events from gas in between BHs / torques on dis



[Armitage et.al;
MacFadyen et.al.;
Dotti et.al;
Chang. et.al.;
Palenzuela et.al.]

Postmerger emission





[Megevand,Anderson,LL,Neilsen;2010]
[Megevand, Anderson,Frank,LL,Liebling,Neilsen,Motl; 2009]

Key qn…

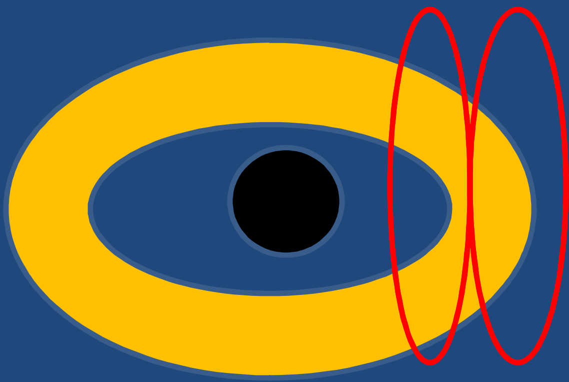
- How are possible jets affected?

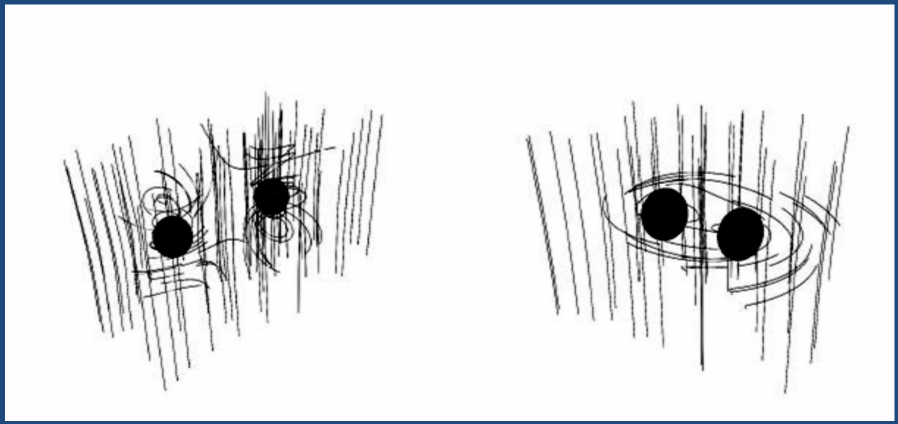
[eg. Hawley,McKinney,Gammie, Stone,Fragile,Komissarov,etc]. Work in progress [Mackinney, LL]

Binary black holes as blenders: 'B-Z' for binaries

How does the curvature influence EM fields?

- Ingredients: GR + Maxwell Eqns (ElectroVac)
  - Binary black holes interacting with magnetic field anchored at a circumbinary disk





[Palenzuela, LL, Anderson,Liebling, Neilsen, PRL 2009 ;
Palenzuela,LL,Yoshida 2009;
Moesta,Palenzuela,Yoshida,Rezzolla,Pollney,LL, submitted]

Approach: Electrovac (GR+EM)

- Einstein equations
- Maxwell equations: now with currents (Force free approx)

Initial setup

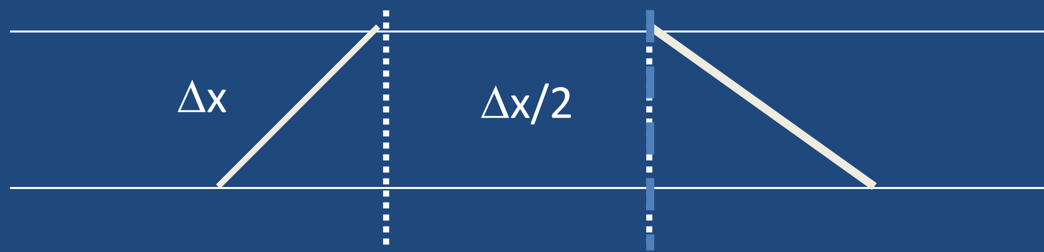
- Quasicircular, equal mass
- Magnetic field as given by a circular loop at far distances ~ constant within computational domain
- Field strength (quite conservative) = 10^4G
  - For this value, if $M_T=10^8 M_O$, EM Energy dens ~ 10^{-16} [$1/M^2$]

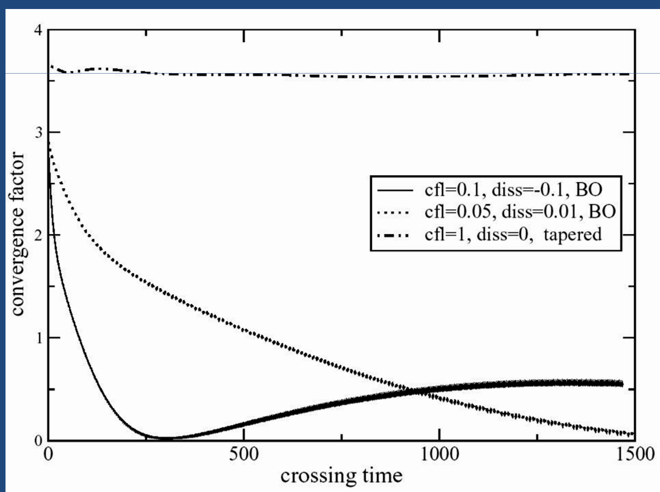
    → EM fields won't affect binary dynamics, but the other way around

    → results hold for fields ~ 10^{10}G. These are too large, though plenty of energy to go around to pump them up if conditions are right [Price-Rosswog, Anderson et.al.,Giacommazo et.al, Liu et.al]

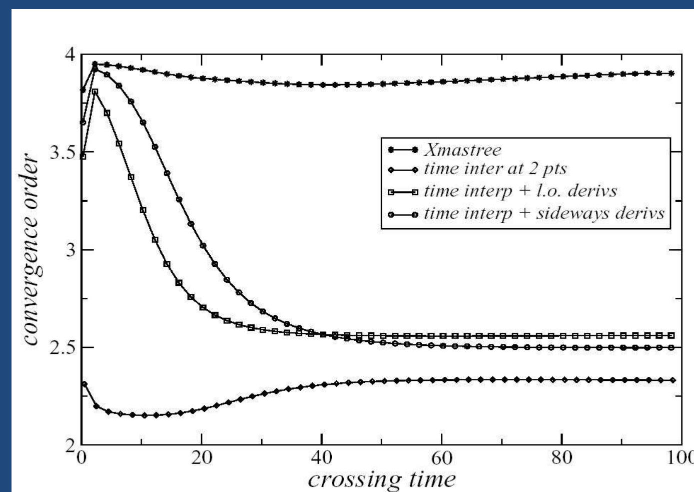
(digress 1) AMR off the way

- Tapered approach [LL,Liebling,Reula]





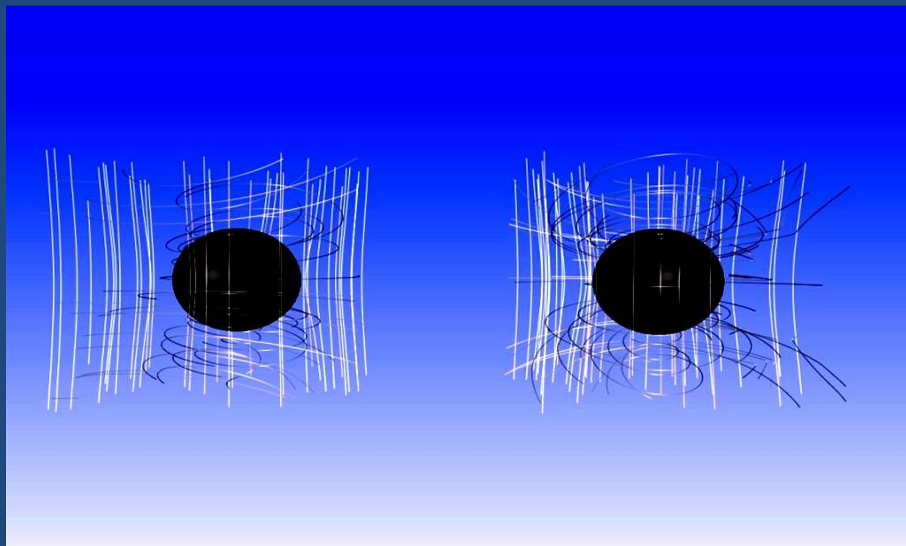
Wave eq

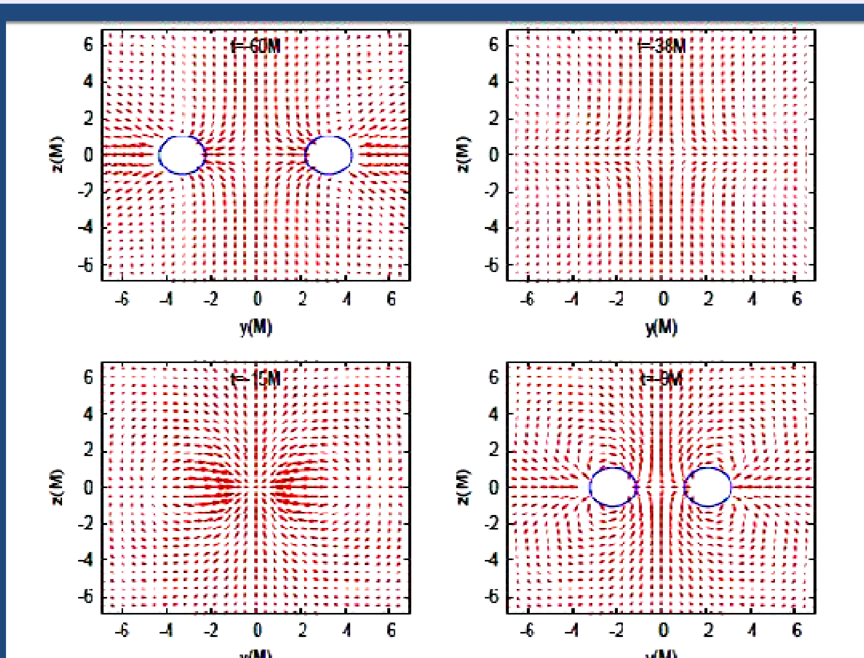


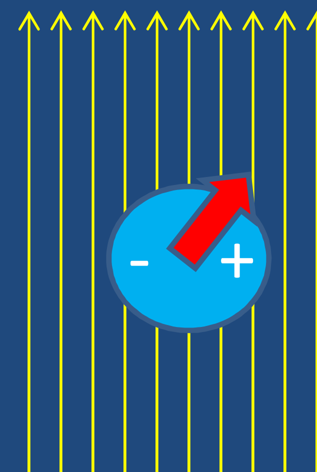
EE

Dynamics…

- At early times, dynamics 'deduced' from membrane paradigm







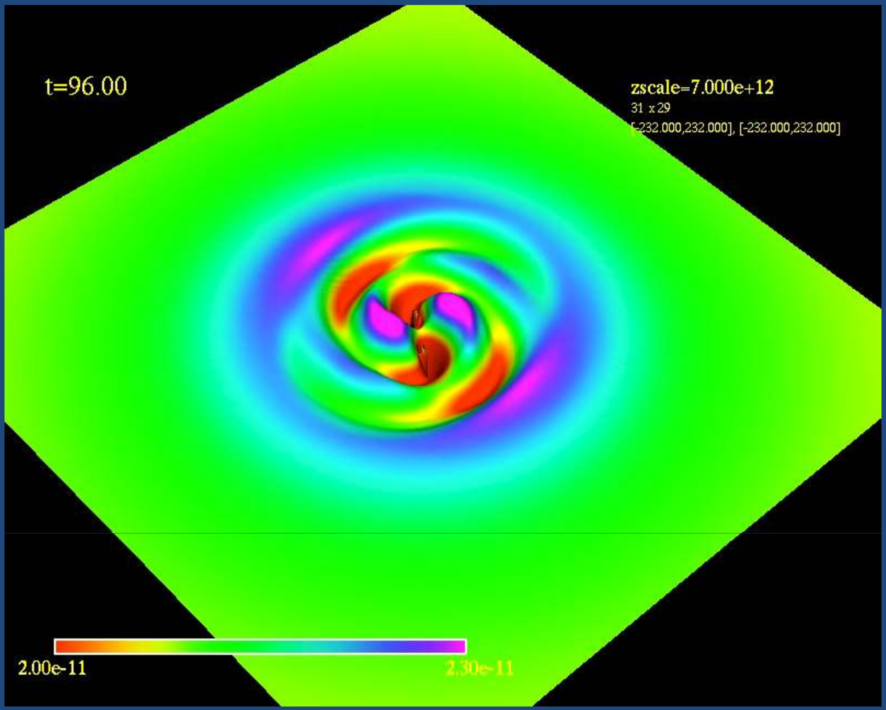
Radiation →

2 dipoles in a circular trajectory

$E \sim v^3$; $B \sim B_o + v^3$; Flux $\sim v^4$

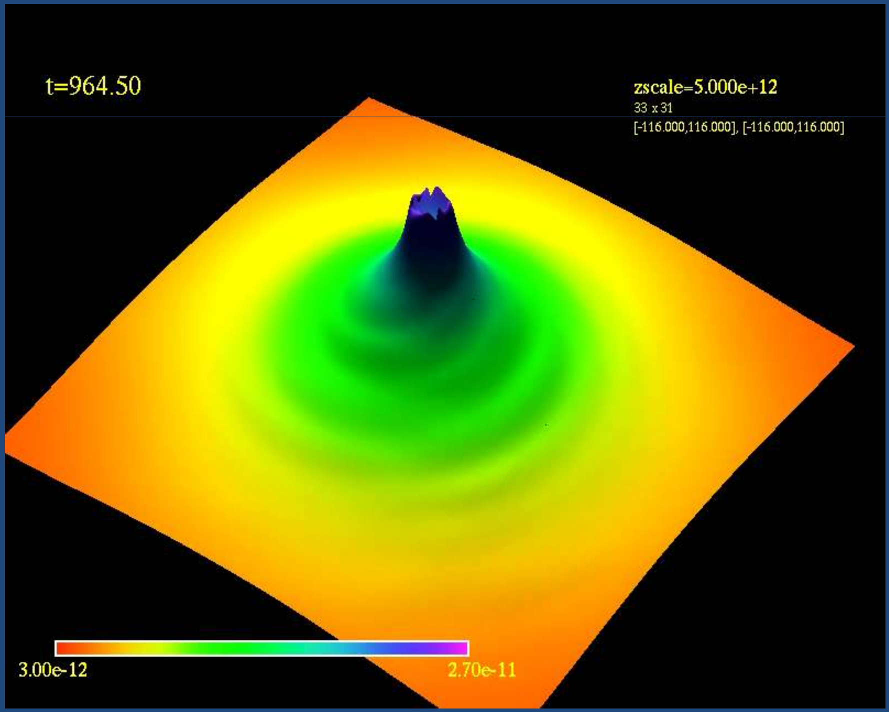
Quadrupole GW $\sim v^4$

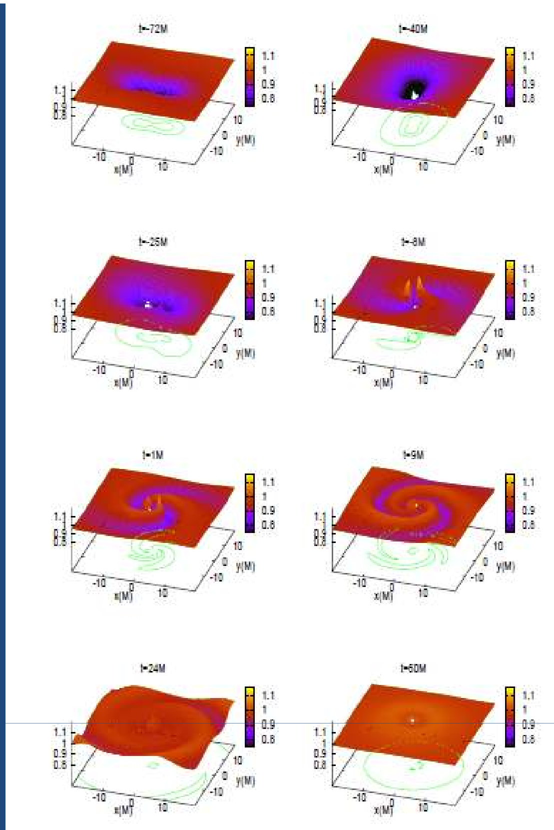
Merger phase

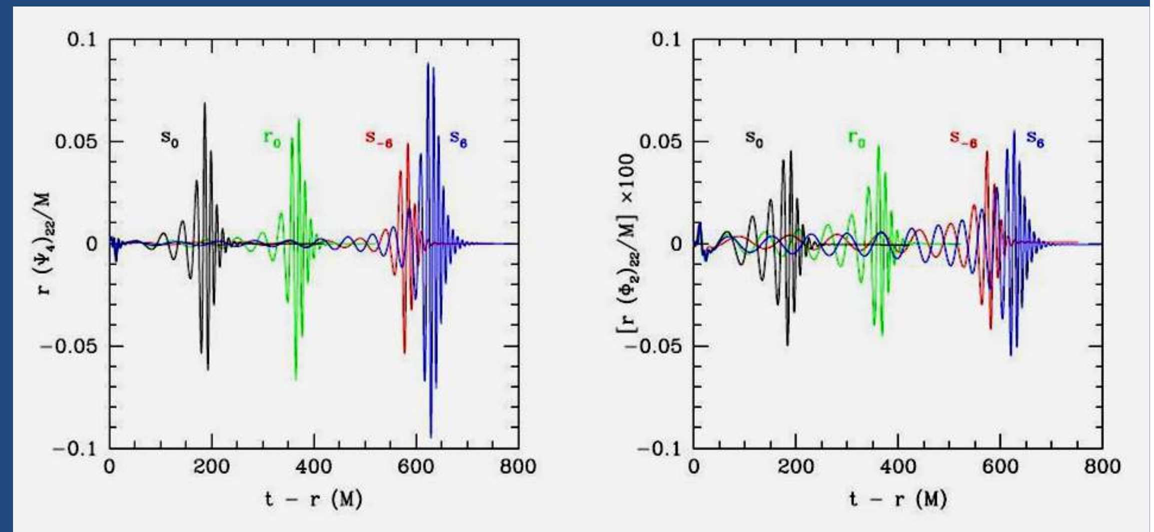


'radial' Poynting

EM Energy density







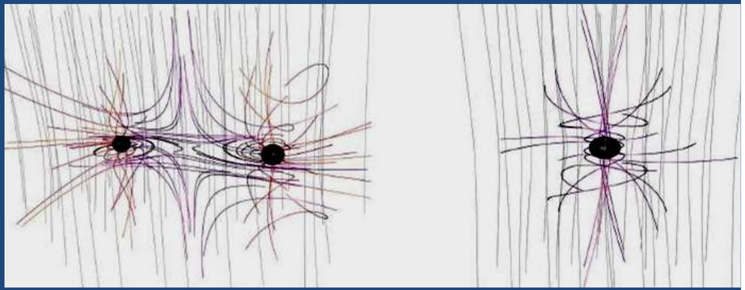
- Time variability given by orbital motion: 2 dipoles orbiting each other (l=2, m=0 mode). E, B vary as GWs, Poynting 'flux' ½ period. (EM tracers of spacetime?)

- Transition through merger gives rise to l=2,m=2 mode.
  - Induced toroidal electric field
  - $E_T \sim (v_{orbital}/c)\, B_z$ ➔ Blandford-Znajek analog….?

Circumbinary disk (pre/prompt)

- Interactions with accretion disk via magnetic fields [GR+ Maxwell]
  - 'Long range' interactions tied by magnetic fields anchored at the disk, reaching the binaries.
  - Binary induces :
    - Strong variability in EM fields, topology change, energy enhancement (reconnection driven effects?)
    - net EM flux can interact with disk , affect accretion rate and emissions by disk. [possible observation needs quiet scenario]
    - Blandford-Znajek (of binary) type mechanism possible though spins required from individual black holes will be high. (spins & B amplitude?)

$E_{EM} \sim E_{GW} \times 10^{-13} (M/10^8 Mo)^2 (B/10^4 G)^2$



QNS:

- Interaction with residual gas yet unexplored
- Individual BZ mechanism?
- Changes due to mass ratios

NS-NS

- Excellent sources of GWs; possible central engine of short GRBs
- Magnetic fields ~ 10^{12} G
- GR simulations available, though uncertainties on:
  - eqn of state
  - Role of magnetic fields (err… neutrinos?)

- For grav waves.
  - Early pre-merger stages: PN is good enough
  - Late pre-merger: careful, internal structure may play a role
  - Merger, postmerger: prompt vs. delayed collapse to a BH and other features, we could use to determine eqn of state.
  - *Can different effects be disentangled?*

- Beyond these, other key qns
  - Does the merger give rise to a BH with sizeable disk?, what is its final spin, magnetic field strength /topology, etc?
  - How long does the hypermassive star exists before collapsing?
  - *All these connect directly with short GRBs models*

NS-NS.

- Einstein equations
  - Generalized Harmonic formulation: $\nabla^a \nabla_a x^u = H^u$
  - Constraints : $C_a = \Gamma_a + H_a$
  - Einstein eqns: $R_{ab} = \nabla_{(a} C_{b)} + {}^{TR}T_{ab} + \kappa\{2n_{(a} C_{b)} - g_{ab} n^c C_c\}$
- GRHydro:
  - Eqns determined by: $$\nabla_a T^{ab} = 0 \quad ; \quad \nabla_a(\rho u^a) = 0$$ $$T_{ab} = (\rho_0(1+\varepsilon) + P)u_a u_b + P g_{ab} + F_a^c F_{bc} - \tfrac{1}{4} F^{cd} F_{cd}$$
  - Expressed in terms of *conservative variables, (use of HRSC)*
  - Eqn of state: $P = (\Gamma - 1)\rho_o \varepsilon \qquad (though,\ P = k\rho_0^{\Gamma}\ for\ ID)$
  - ideal MHD limit
    - Constraint (s)?
      - How to deal with it? (eqns weakly hyperbolic if non zero, or something else done)
      - How to not break it when using AMR? → tapered method
      - How to ensure boundaries won't get in the way? → constraint pres. bound cond.
      - How to make all transparent for global treatment of the problem?

(digress 2) MHD eqns. Simple example

- $\alpha = 0$: conservative, but weakly hyperbolic
- $\alpha = 1$: non-conservative, but hyperbolic!
- c_l ; s drivers of constraint

$$\partial_t \rho = -\nabla_i(\rho v^i)$$
$$\rho\partial_t v^i = -\rho v^j \nabla_j v^i - \nabla^i p - B_k(\nabla^i B^k - \nabla^k B^i) - \alpha B^i \nabla_k B^k$$
$$\partial_t B^i = -\nabla_j(u^j B^i - u^i B^j) - \alpha u^i \nabla_j B^j - c_l \nabla^i \phi$$
$$\partial_t e = -\nabla_i((e + p + \frac{1}{2}B^2)v^i - B^i v \cdot B) - \alpha v^i B_i \nabla_k B^k - c_l B^k \nabla_k \phi$$
$$\partial_t \phi = -\alpha u^j \nabla_j \phi - c_l \nabla_j B^j - s\phi$$

where

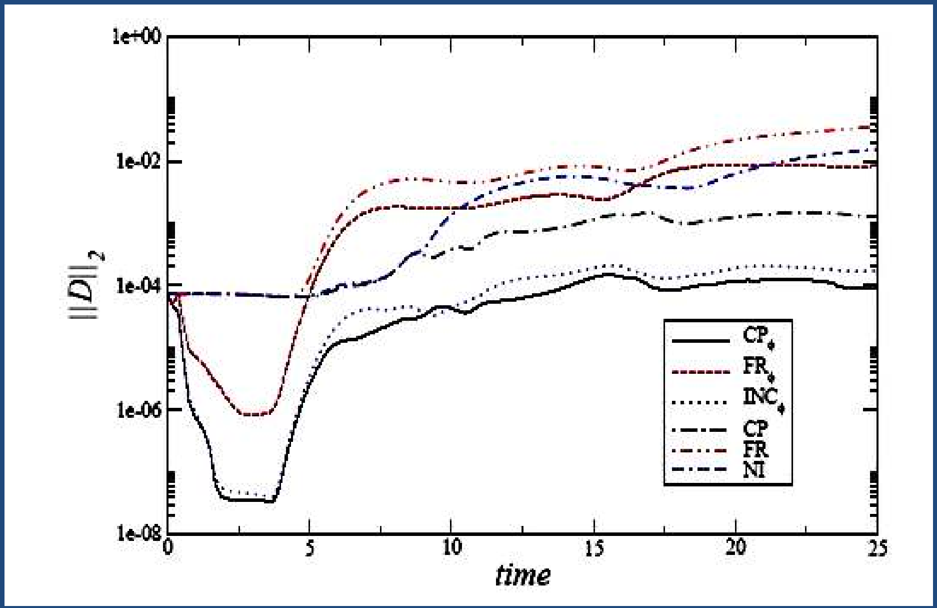
$$p := (\gamma - 1)(e - \frac{1}{2}\rho v^2 - \frac{1}{2}B^2) \qquad c_s^2 := \gamma p/\rho$$

- Strongly/symmetric hyperbolic → complete set of eigenvectors

- Constraint Preserving Bound. Cond.
→induce eqn for boundary values

$$(\nabla B)_{,t} = 0 \rightarrow (\partial_n B)_{,t} = known + bdry\ data$$

[Cecere,LL,Reula 07]

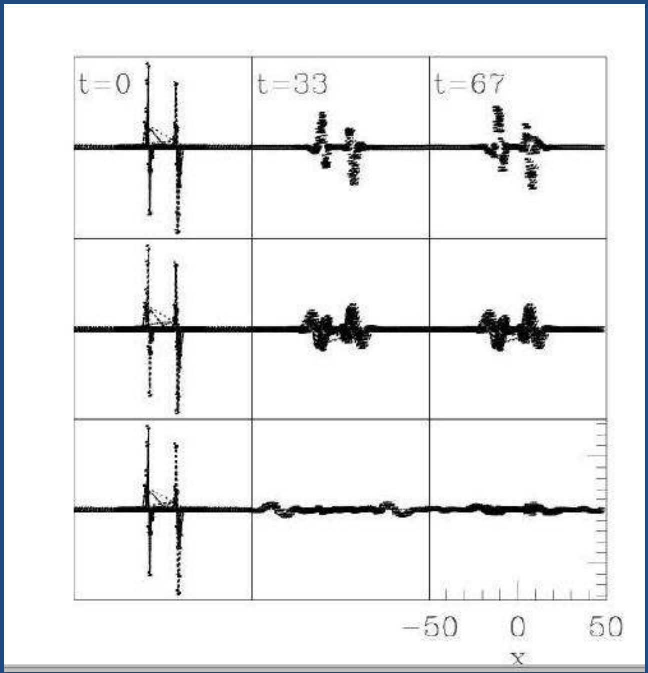


[Cecere,LL,Reula]

Constraint control

Boundary conditions

Flexibility of picking algorithms (eg. not tied to constraint transport)



GRMHD. Star + by-hand violation

[Liebling,LL,Neilsen,Palenzuela]

(digress 3) Last rabbit out of a hat

- ‘Extended’ ideal MHD equations

- Terms added for :
  - Divergence cleaning (c_r , c_h)
  - Ensure strong hyperbolicity eve if no div cleaning used.
  - if no added field, ‘eight’ wave formulation. (but has sources with derivatives)

$$\partial_t \tilde{D} + \partial_i \left[\alpha \tilde{D} \left(v^i - \frac{\beta^i}{\alpha}\right)\right] = 0,$$

$$\partial_t \tilde{S}_j + \partial_i \left[\alpha \left(\tilde{S}_j \left(v^i - \frac{\beta^i}{\alpha}\right) + \sqrt{h}\, P\, h^i{}_j\right)\right]$$

$$= \alpha\, {}^3\Gamma^i{}_{jk} \left(\tilde{S}_i v^k + \sqrt{h}\, P h_i{}^k\right) + \tilde{S}_a \partial_j \beta^a$$

$$-\partial_j \alpha \left(\tilde{\tau} + \tilde{D}\right)$$

$$-\zeta \alpha (\tilde{B}_i W^{-2} + v_i v_j \tilde{B}^j) \partial_k \tilde{B}^k,$$

$$\partial_t \tilde{\tau} + \partial_i \left[\alpha \left(\tilde{S}^i - \frac{\beta^i}{\alpha} \tilde{\tau} - v^i \tilde{D}\right)\right]$$

$$= \alpha \left[K_{ij} \tilde{S}^i v^j + \sqrt{h}\, K P - \frac{1}{\alpha} \tilde{S}^a \partial_a \alpha\right],$$

$$-\zeta \alpha v_j \tilde{B}^j \partial_k \tilde{B}^k$$

$$\partial_t \tilde{B}^b + \partial_i \left[\tilde{B}^b \left(v^i - \frac{\beta^i}{\alpha}\right) - \tilde{B}^i \left(v^b - \frac{\beta^b}{\alpha}\right)\right]$$

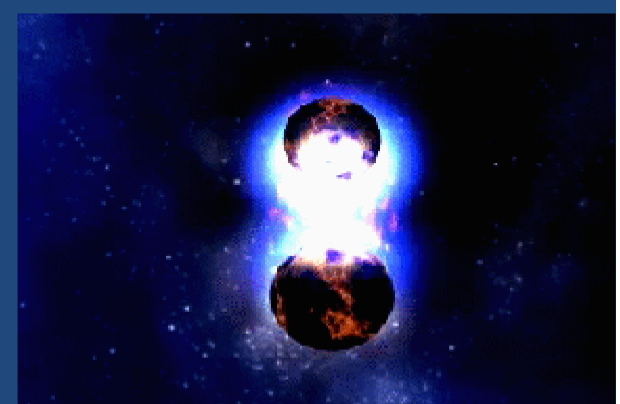
$$= -h^{bj} \partial_j \Psi - \alpha \sqrt{h} h^{ij} \partial_j \Psi - \zeta \alpha v^i \partial_j \tilde{B}^j$$

$$\partial_t \Psi = -c_r \alpha \Psi - c_h \frac{\alpha}{\sqrt{h}} \partial_i \tilde{B}^i + (\beta^i - \alpha v^i) \partial_i \Psi$$

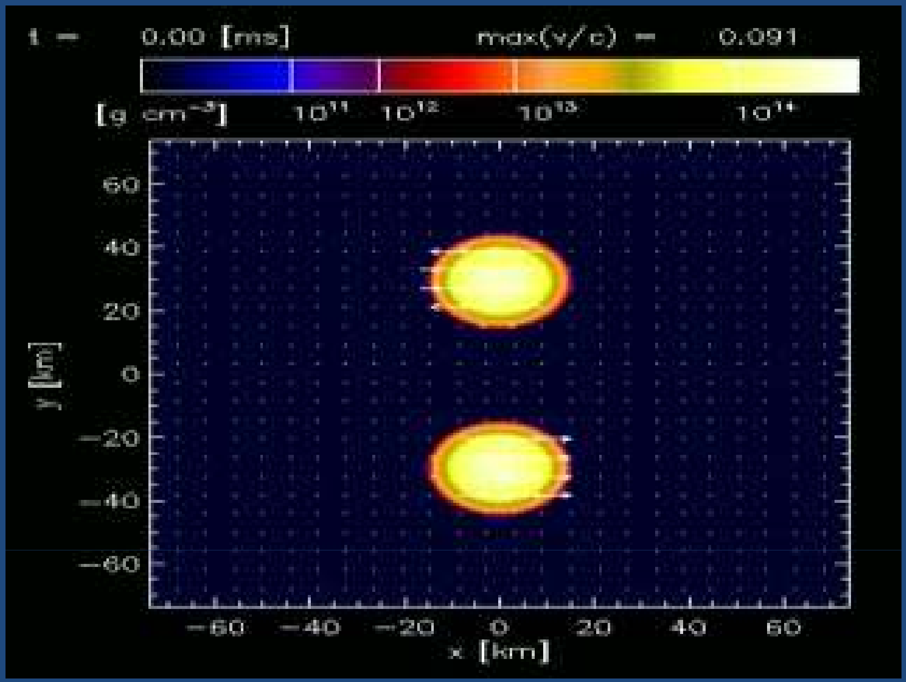
Initial configuration. (Not ‘too physical’…)

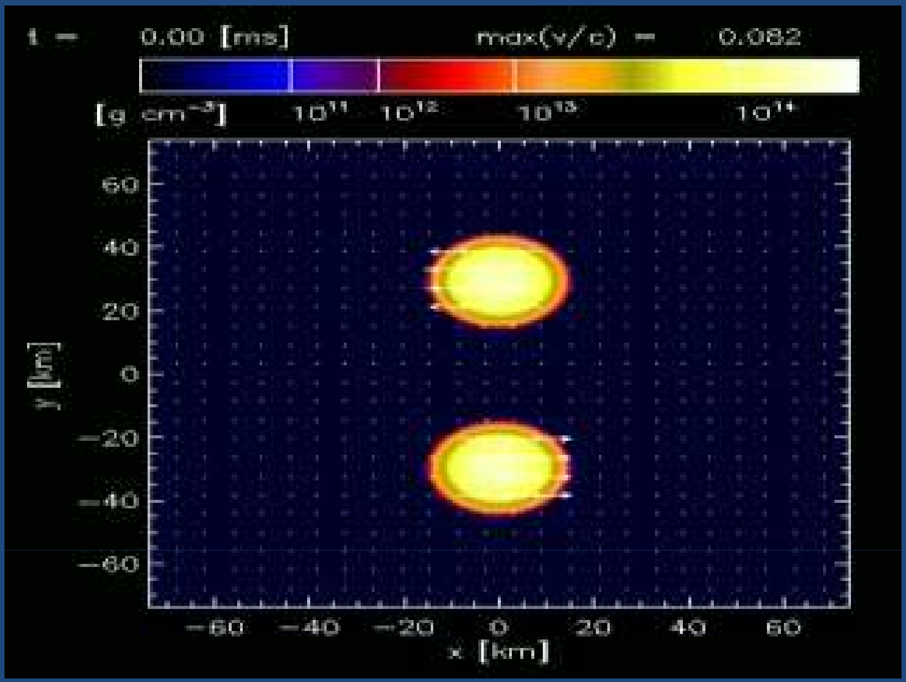
- Equal non-rotating polytropes to represent the stars (Γ=2). (R_s=16.26km, M~0.9 M_O)

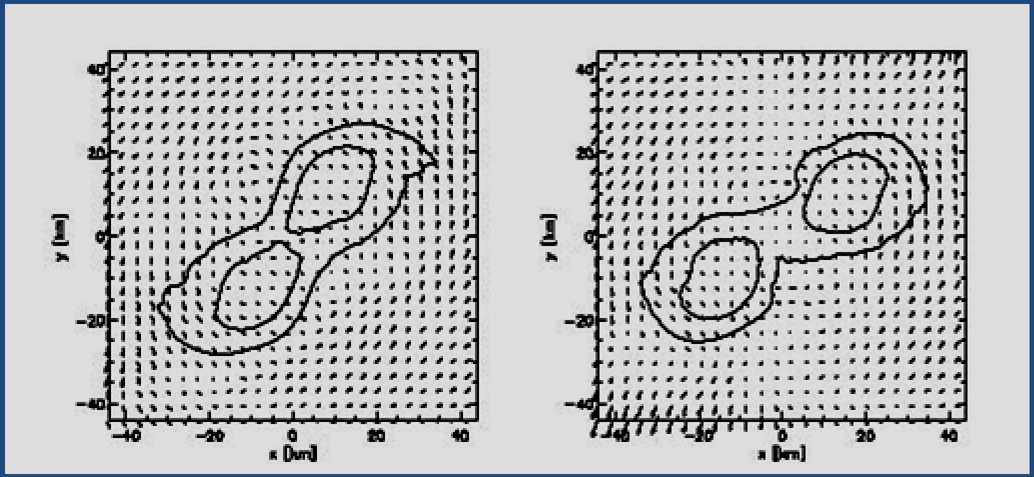
- Poloidal seed magnetic fields, antialigned with orbital angular momentum in each star. B ~ 9 10^{15} G
- Initial separation ~ 4 R_s
- Grid : [-100R_s , 100R_s]; up to 7 levels of refinement Δ_{min}=0.46km. Gravitational waves extracted well within the wave zone.

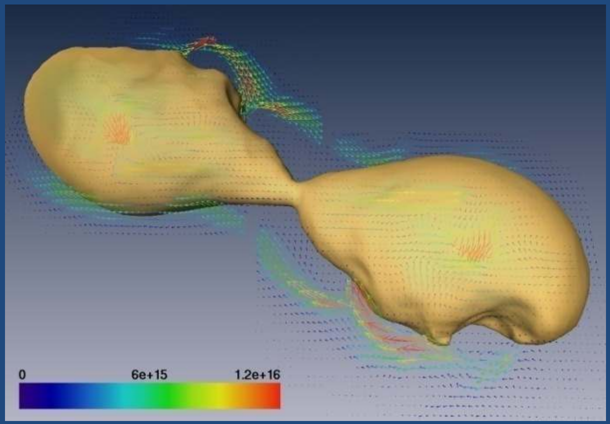


[Anderson,Hirschmann,LL,Motl,Neilsen,Palenzuela,Tohline 08]

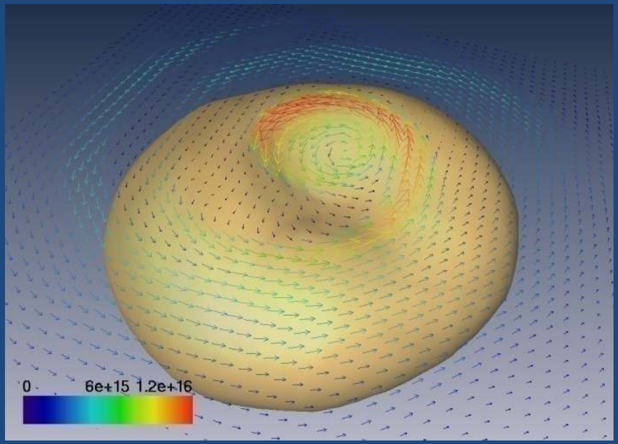

t = 0.00 [ms]
max(v/c) = 0.091
[g cm⁻³]
10¹¹ 10¹² 10¹³ 10¹⁴
y [km]
x [km]
60 40 20 0 −20 −40 −60


t = 0.00 [ms]
max(v/c) = 0.082
[g cm⁻³]
10¹¹ 10¹² 10¹³ 10¹⁴
y [km]
x [km]
60 40 20 0 −20 −40 −60

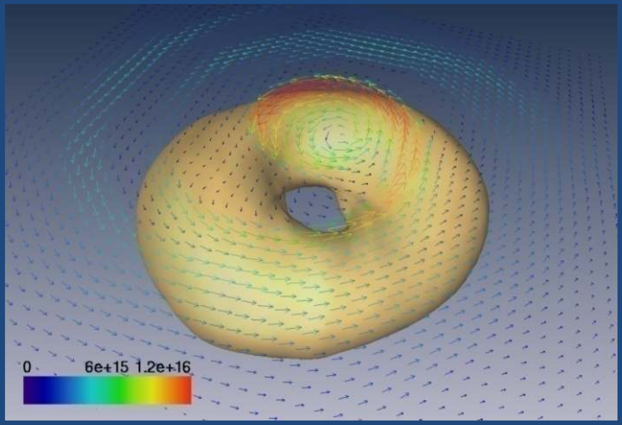

y [km]
x [km]
40 20 0 -20 -40



ρ=10^{14}g/cm^3



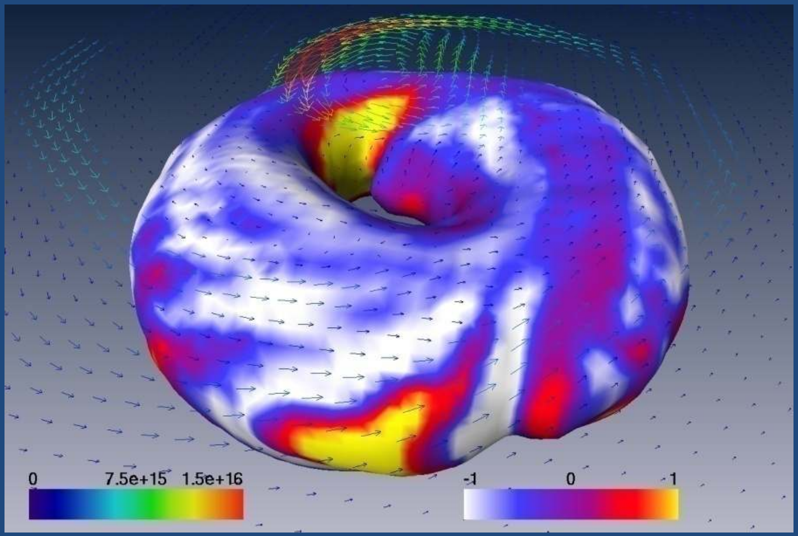
ρ=8 10^{13}g/cm^3



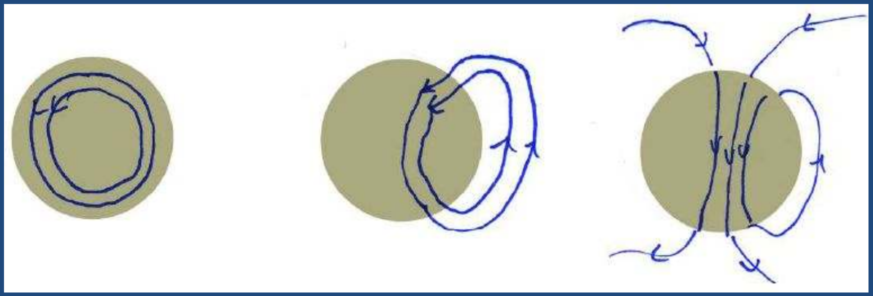
ρ=10^{14}g/cm^3

- K-H instability, shear energy into magnetic fields which grow ~ 1 order of magnitude. Growth saturates ~ Alfven time-scale
- Hypermassive star differentially rotating at rates different from the non-magnetized case

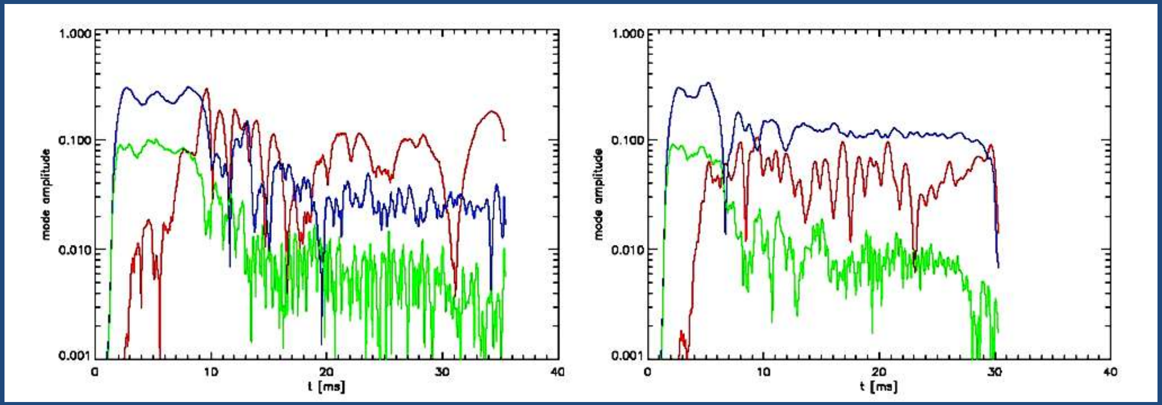
Less obvious effects



Magnetic Buoyancy

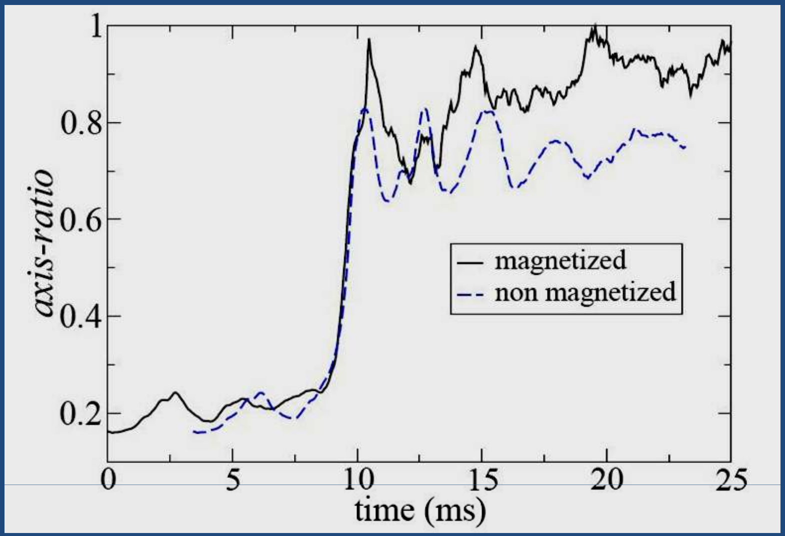


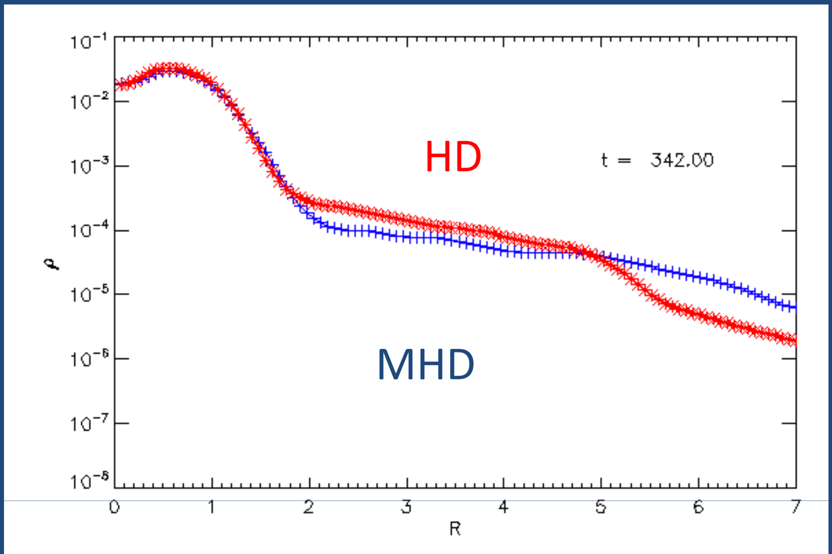
Ruderman-Kluzniak
B ~ 10^{17}G

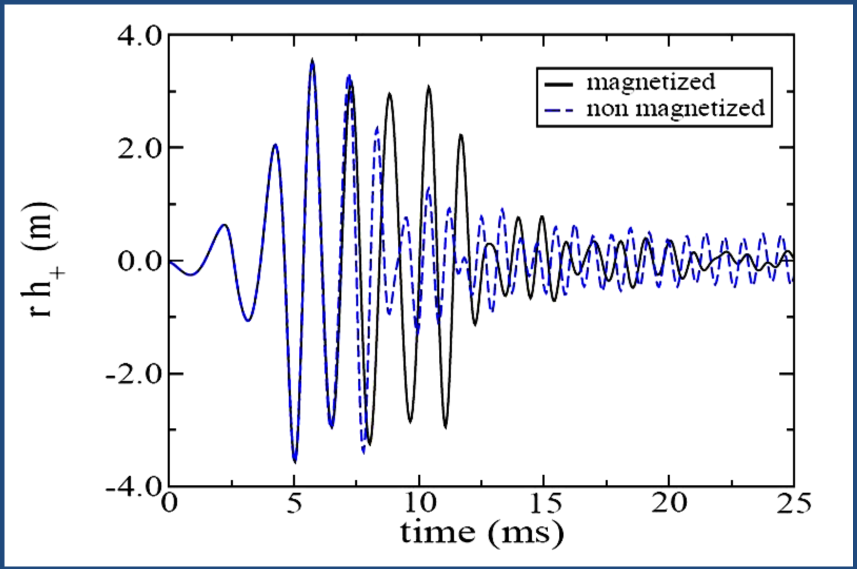


l = 2 decreasing consistently with gravitational wave emission rate
l = 4 ‘Cartesian grid’ induced remains under control throughout
l = 1 mode growing, consistent with Tayler’s instabilty (magnetic field)

‘bulk’ dynamics & consequences







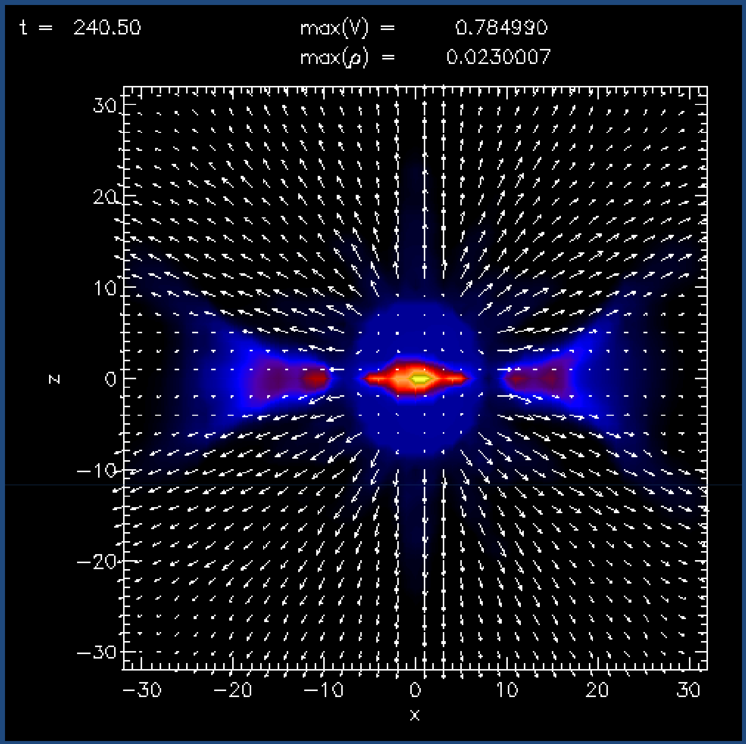
Amplitudes ~
Consistent with noise level to 2-4Mpc in current LIGO

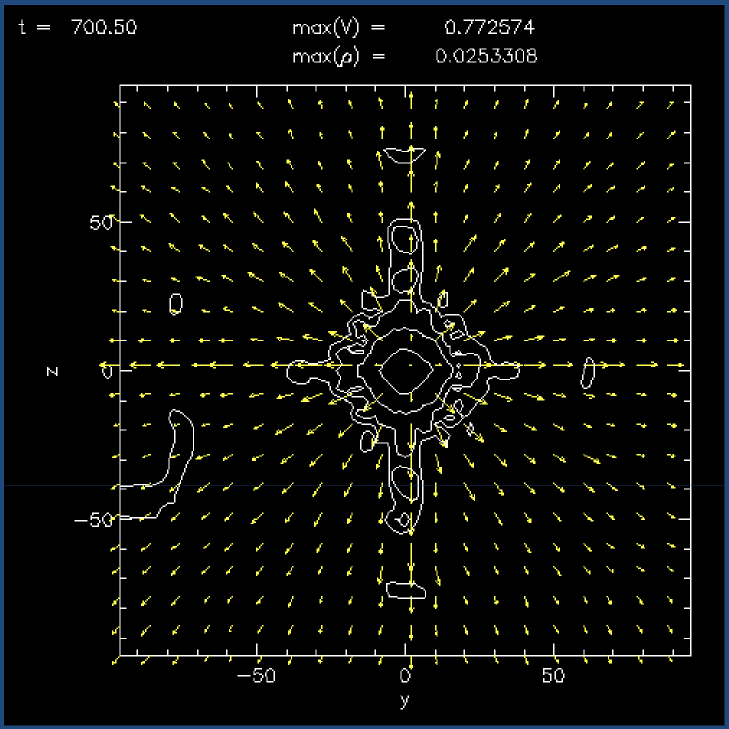
Matches:

- 0.98 to t=7.5ms
- 0.63 to t= 12.5ms
- 0.59 to t=25ms

[related to Price-Rosswog 06]

Other hints…



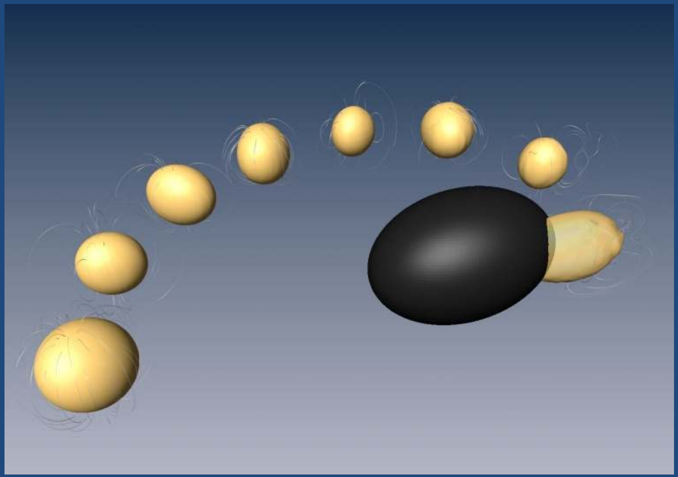


- Could these induce asymmetric bubbles? [Bucciantini et. al.]
- how about long-term behavior of the disk? Other models will need to take over [Metzger et.al.]
- *No right to expect this would work! Not the right approx to study the different regimes*

as opposed to BH-BH case, too early to hand it over....

BH-NS

- GR studies ongoing (NS: polytrope or via Shen eos)
  - Sizeable disks if sufficiently high spins (>~ 0.5)
  - 'competition' with mass ratio
  - Magnetic field effects?



Neutron Star: Irrotational, $\Gamma = 2$
R = 15 [km] M = 1.4 $M_{solar.}$

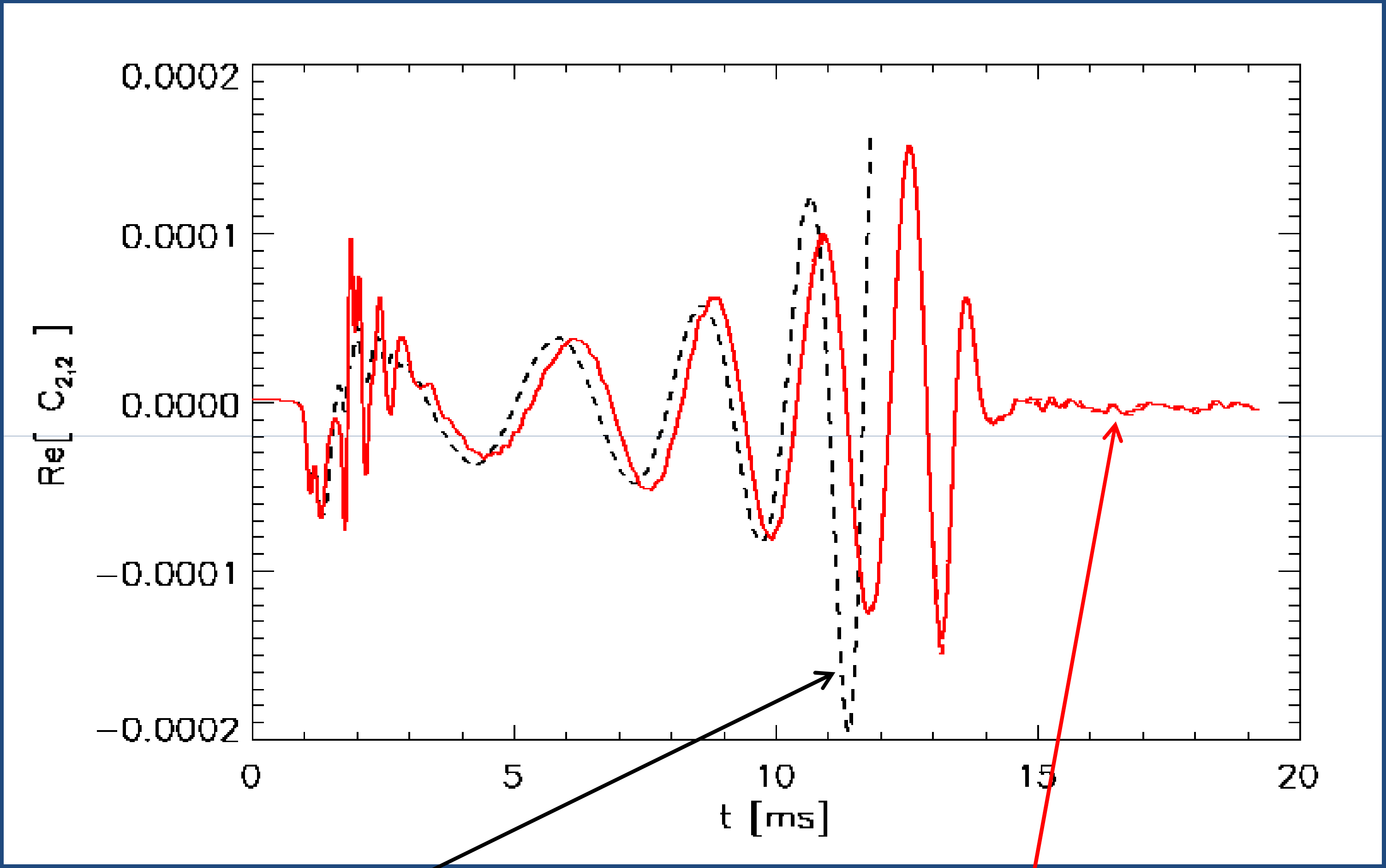
Initial dipole field of strength 10^{12} [Gauss]

Black Hole:M = 7 $M_{solar\,;}$ a/M = 0, 0.5

Initial separation of 100 [km]
Grid extends to ± 443 [km]
Peak resolution of 0.73 [km] or 40 points across initial neutron star

[P.Motl, M.Anderson, M. Besselman, S. Chawla, LL, S.Liebling, D. Neilsen, J.Tohline]

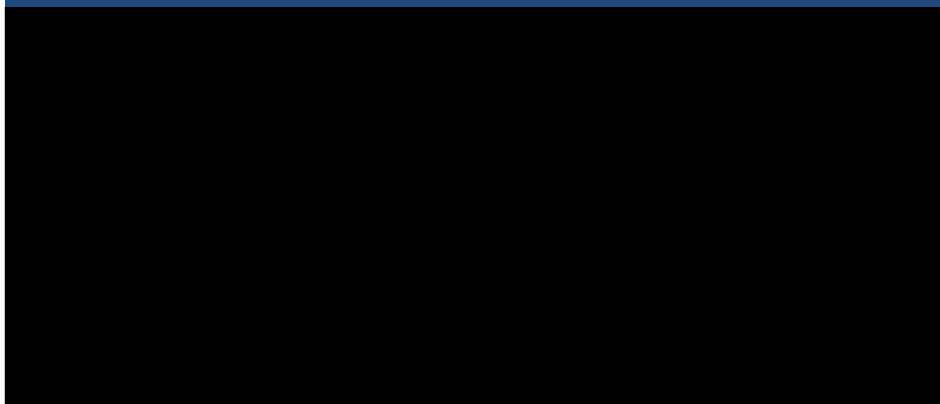
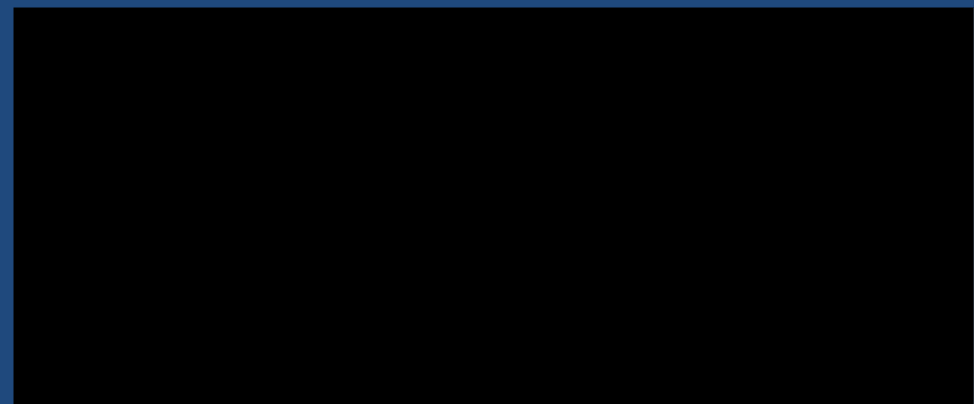
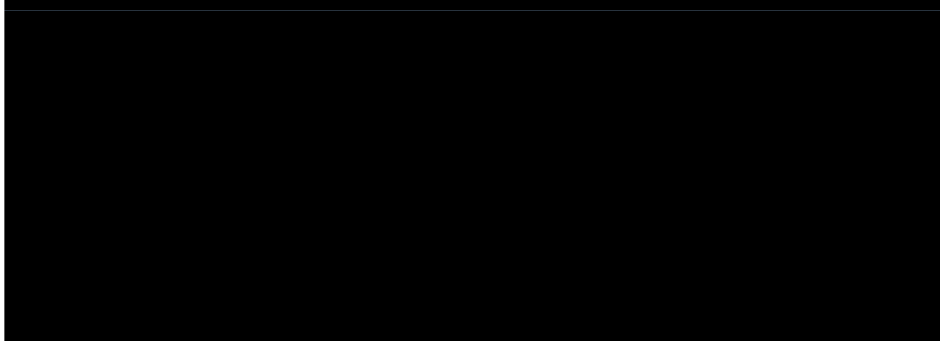
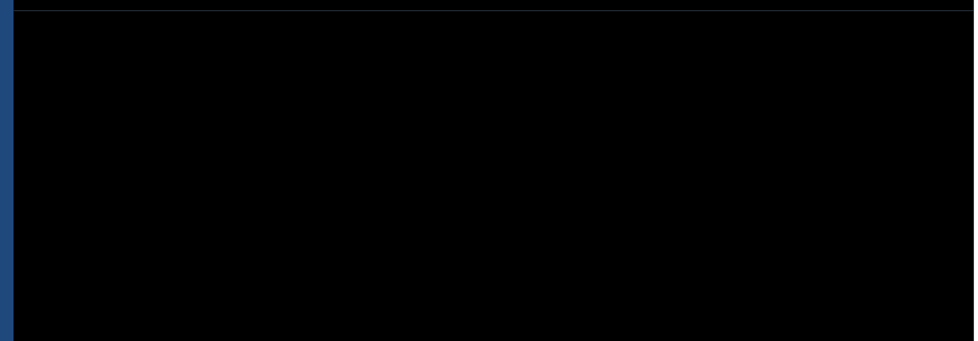
Grav. Waves…



a = 0, B = 10^{12}

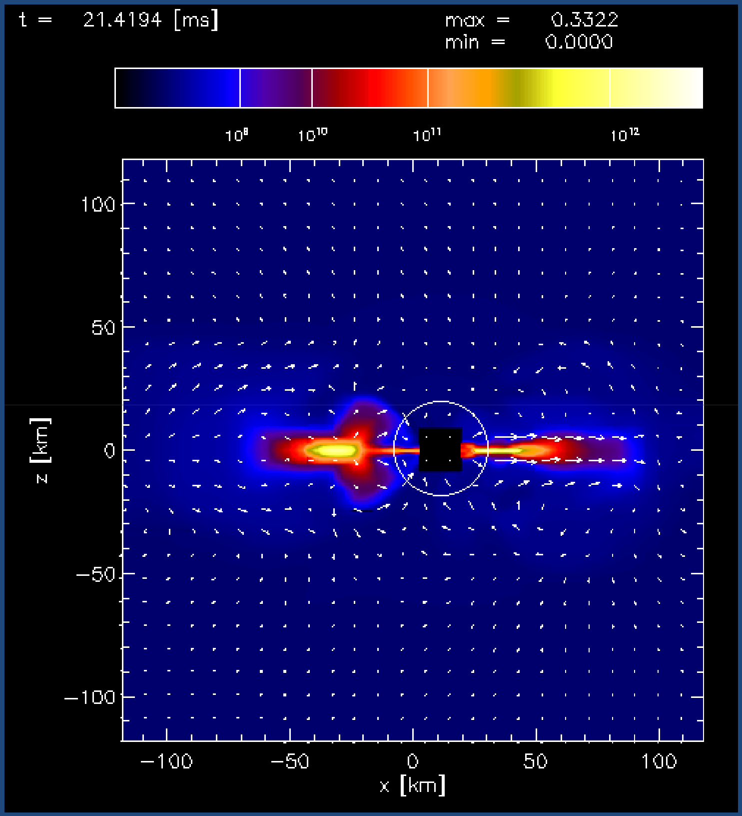
a = 0.5, B = 0 and
a = 0.5, B = 10^{12}

Different cases…

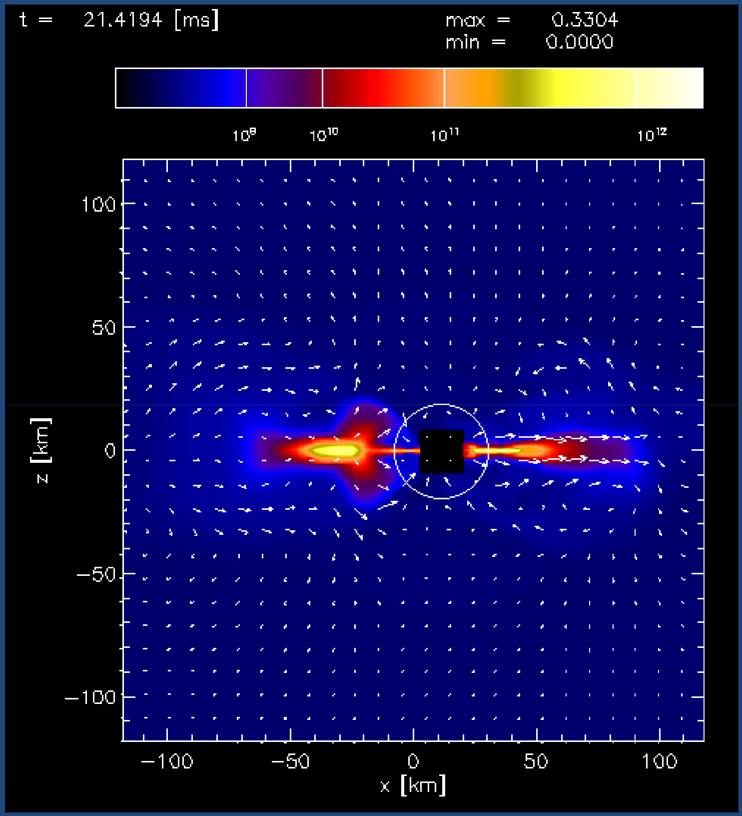





Disk and structure (a/M=0.5)

Unmagnetized



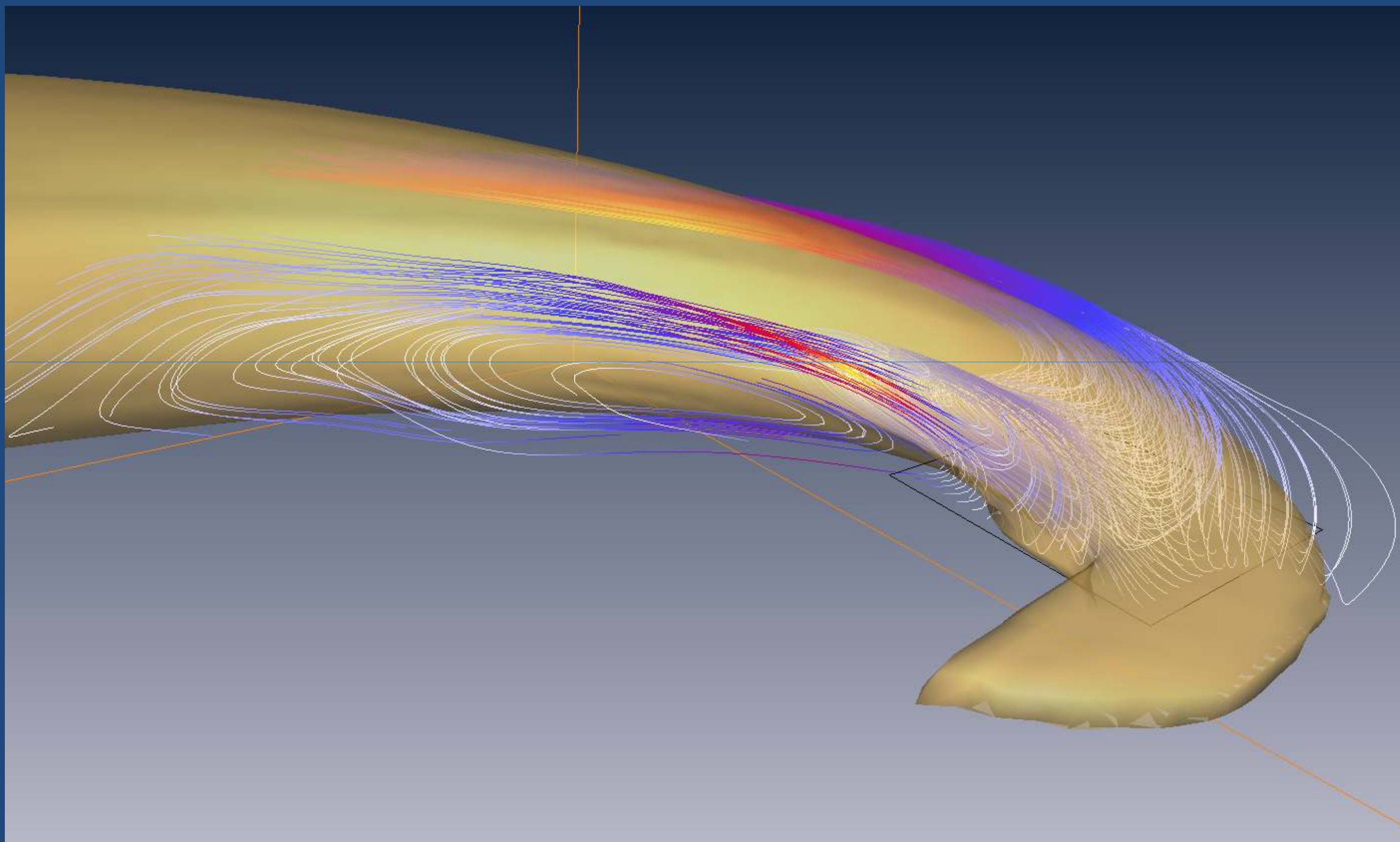
$B = 10^{12}$ initially



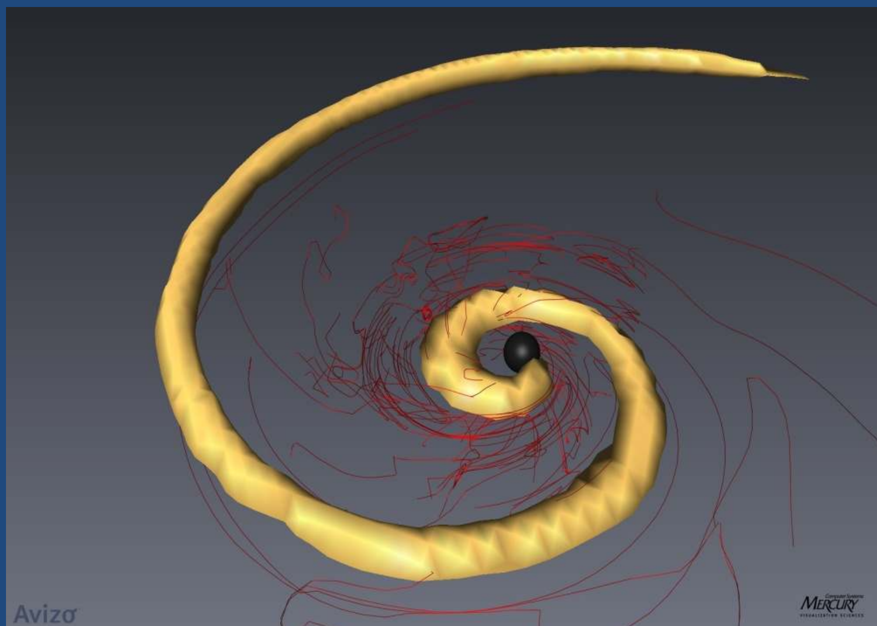
$M_{Disk} = 1.7\%$

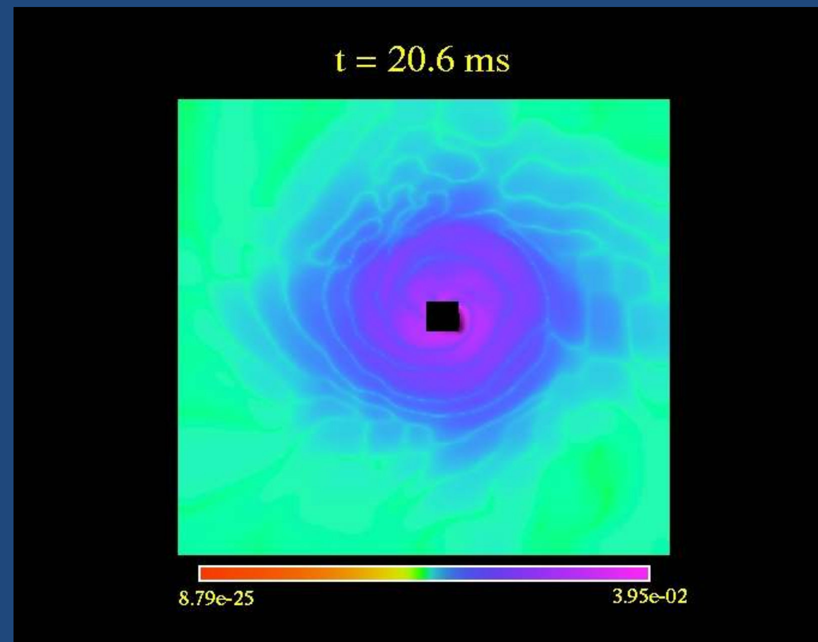
$M_{Disk} = 1.6\%$

Field lines & behavior



Handing it over…





[t=15.6 ms]
→ Going toroidal…

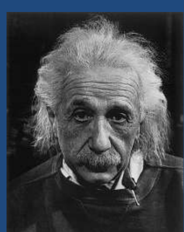
[t=20ms]
B ~ 10^{12} → 10^{13}G

--Long times…. Radiation transport effects to be accounted for yet
--connection with fall back model for SGRB's tails [Rosswog,Lee&Ramirez Ruiz]

Final words

- Interesting problems can be studied with different approx and connect with other studies

- Full problem requires ability to study several regions at the same time that will change dynamically.
  - Building up a new approach. GR + Maxwell + Fluid eqns with suitable model for currents, lying half way in between:



    - Novel, unorthodox approach through the path less taken
    - if brute force doesn’t work… use more of it!

Talk by Palenzuela on Saturday.