Introducing Flow-er: a Hydrodynamics Code for Relativistic and Newtonian Flows

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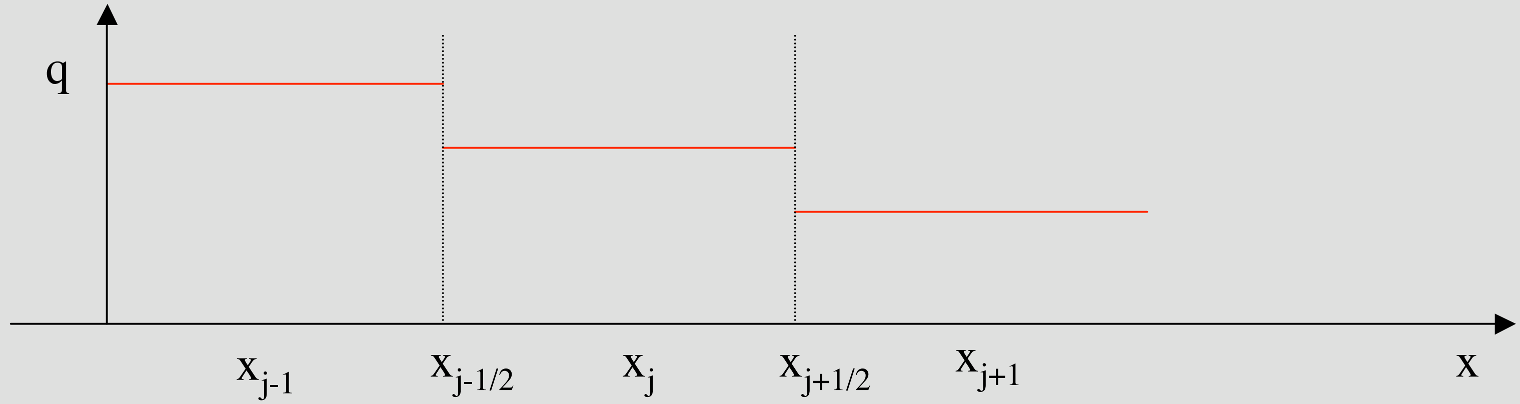
Flow-er Features

- Implements equations for Newtonian and general relativistic hydrodynamics
- One, two or three spatial dimensions
- Based on the Kurganov & Tadmor, non-oscillatory central difference (NOCD) scheme
- Time integration with Shu & Osher's R-K 3
- Choice of six reconstruction schemes
- Choice of second or fourth order finite difference and interpolation operators
- One of the physics packages within the HAD adaptive mesh refinement toolkit

Flow-er Features: Kurganov - Tadmor Algorithm

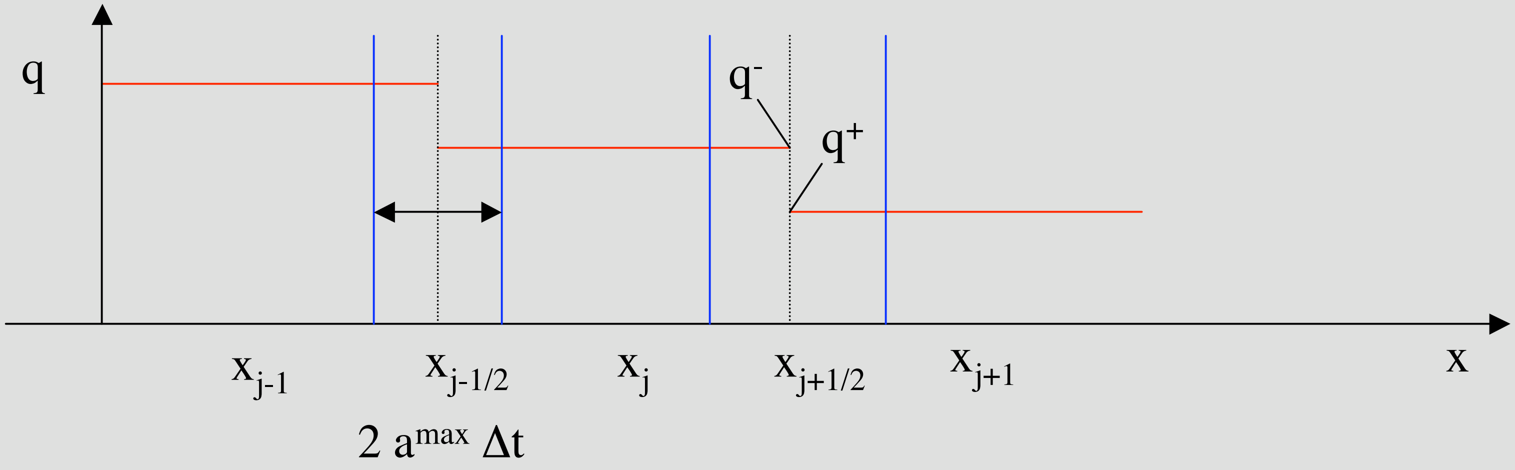


In a Godunov view, cell interfaces look like Riemann problems. Use the Riemann solution to compute fluxes at $\{x_{j+1/2}\}$ to update values at $\{x_j\}$

In central schemes, update the value at staggered grid locations, $\{x_{j+1/2}\}$ in terms of fluxes at $\{x_j\}$

Because of the Courant condition, we know that the discontinuities can have no influence there. Then remap the solution back to the original grid.

Flow-er Features: Kurganov - Tadmor Algorithm



The Kurganov-Tadmor scheme (2000 JCompPhys 160 p241) refines this idea further by limiting the region of influence from the discontinuities. The scheme that results upon taking the limit of $\Delta t \rightarrow 0$ has a simple form. For a hyperbolic conservation law of the form

$$\partial_t q + \partial_x f(q) = 0$$

The Kurganov-Tadmor scheme is

$$\partial_t q_j = - (1/\Delta x) (F_{j+1/2} - F_{j-1/2})$$

$$F_{j+1/2} = (1/2) (f(q^+_{j+1/2}) + f(q^-_{j+1/2})) - (1/2) a^{max}_{j+1/2} (q^+_{j+1/2} - q^-_{j+1/2})$$

Flow-er Features: Reconstruction Schemes

Linear reconstruction with slopes calculated via:
- the minmod function
- the uniformly non-oscillatory limiter (uno, Harten & Osher 1987) or
- VanLeer slope (e.g. Anninos & Fragile 2003)

Quadratic reconstruction with
- Liu's limiter (Liu & Tadmor 1998) or
- Kurganov's limiter (Kurganov & Petrova 2001)

The Piecewise Parabolic reconstruction of Colella & Woodward (1984) with optional discontinuity detection and dissipation

Flow-er Features: Relativistic Hydrodynamic Equations

3+1 decomposition with equations as in Banyuls *et al*. 1997

$1/\sqrt{-g}\ \partial_t (\sqrt{\gamma}\ D) + 1/\sqrt{-g}\ \partial_i (/\sqrt{-g}\ D\ (v^i - \beta^i/\alpha)) = 0$

$1/\sqrt{-g}\ \partial_t (\sqrt{\gamma}\ S_j) + 1/\sqrt{-g}\ \partial_i (/\sqrt{-g}\ \ (S_j\ (v^i - \beta^i/\alpha)\ + P\delta^i_j)) = T^{\mu\nu} (\partial_\mu g_{\nu j} - \Gamma^\lambda{}_{\mu\nu}\ g_{\lambda j})$

$1/\sqrt{-g}\ \partial_t (\sqrt{\gamma}\ \tau) + 1/\sqrt{-g}\ \partial_i (/\sqrt{-g}\ \ (\tau\ (v^i - \beta^i/\alpha)\ + P\ v^i)) = \alpha (T^{t\mu}\ \partial_\mu \ln \alpha - T^{\mu\nu}\ \Gamma^t{}_{\mu\nu})$

Where

$D = \rho\ W$

$S_j = \rho\ h\ W^2\ v_j$

$\tau = E - D = \rho\ h\ W^2 - P - D$

$W^2 = (1 - v_i v^i)^{-1}$

$h = 1 + \varepsilon + P / \rho$

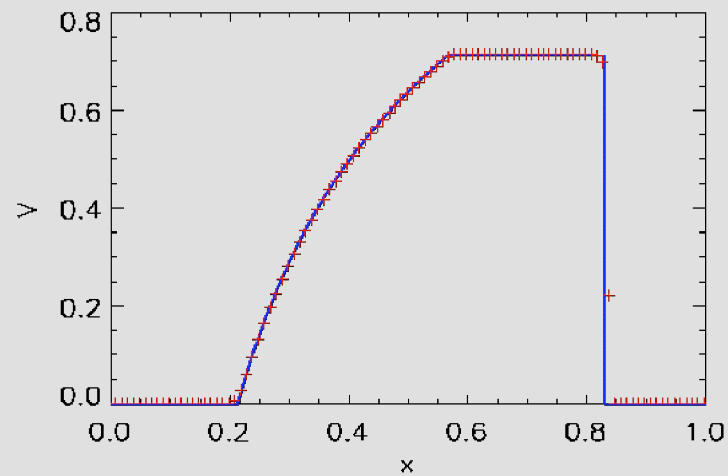
$P = (\Gamma-1)\ \rho\ \varepsilon$

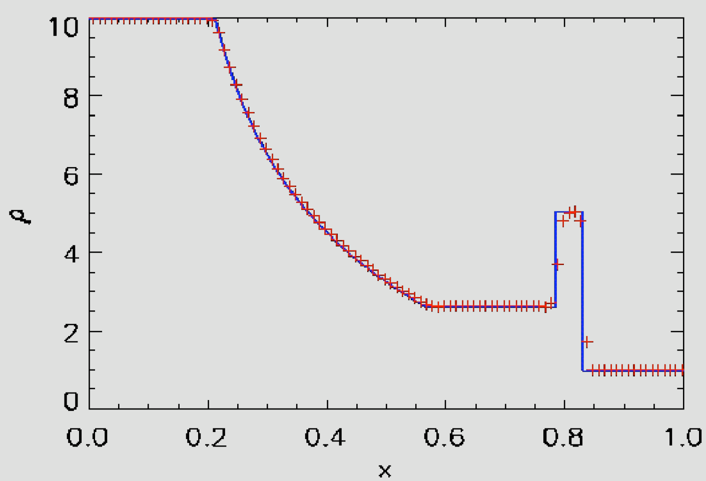
Recovery of primitive variables from the conserved quantities is accomplished with Eric Hirschman and David Neilsen's RMHD solver in the limit of no magnetic field (gr-qc/0512147).

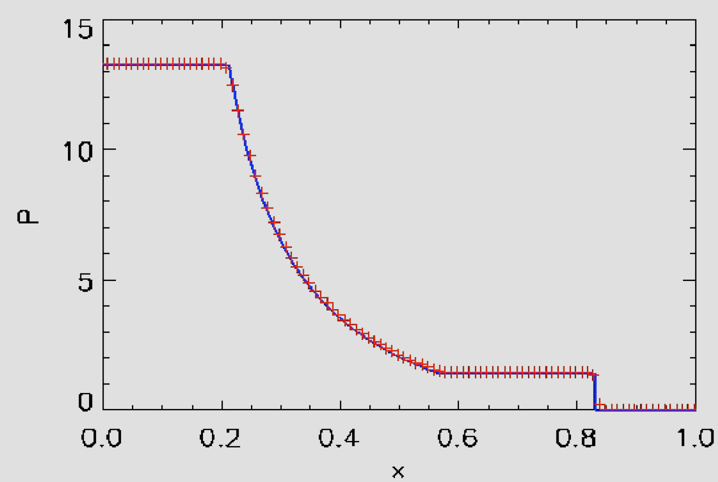
Special Relativistic Test Problems: Moderate Shock Tube

Ideal gas with $\Gamma = 5/3$
Left state: $\rho = 10$, $P = 13.3$, $v = 0$
Right state: $\rho = 1$, $P = e\text{-}6$, $v = 0$

Flow with Lorentz factor of 1.43

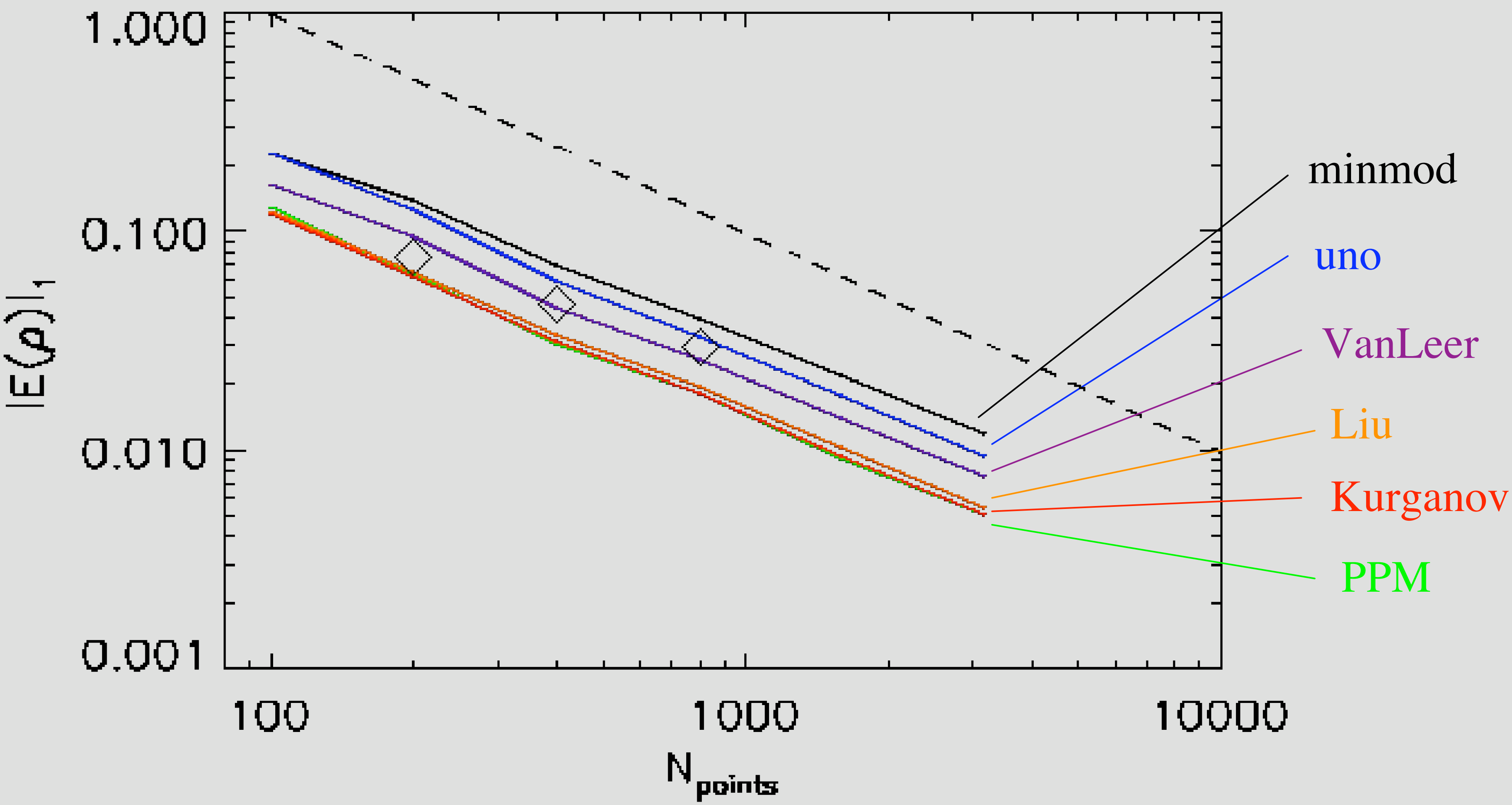






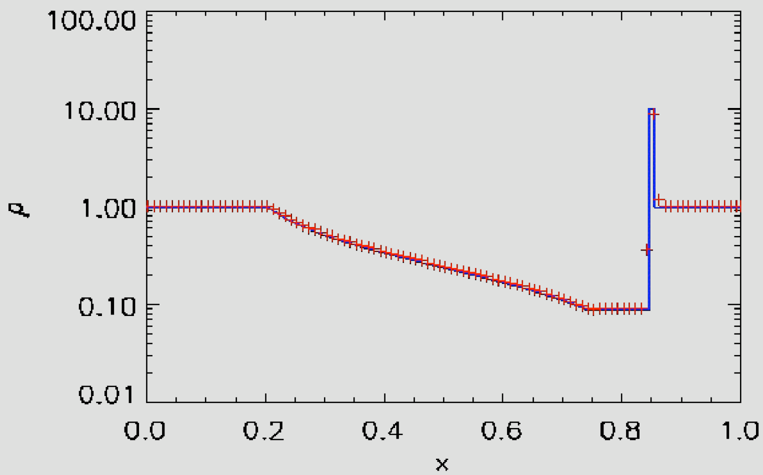
Solution at $t = 0.4$ with Kurganov reconstruction and 400 points

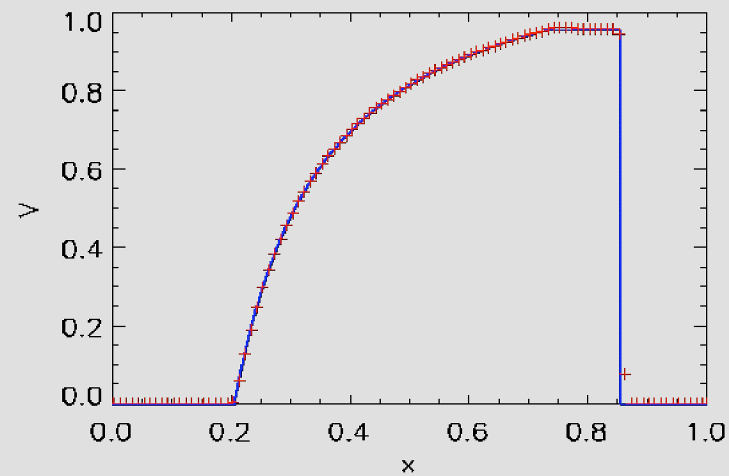
Special Relativistic Test Problems: Moderate Shock Tube



Diamond points are from Font *et al*. 2000 using the Marquina flux formula

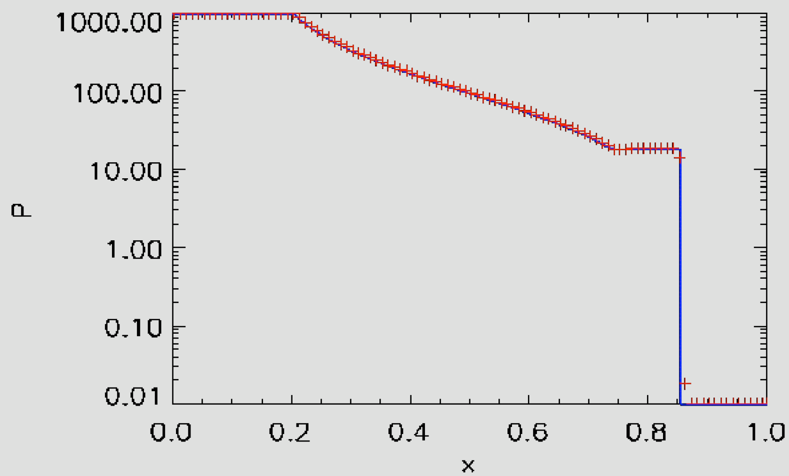
Special Relativistic Test Problems: Stronger Shock





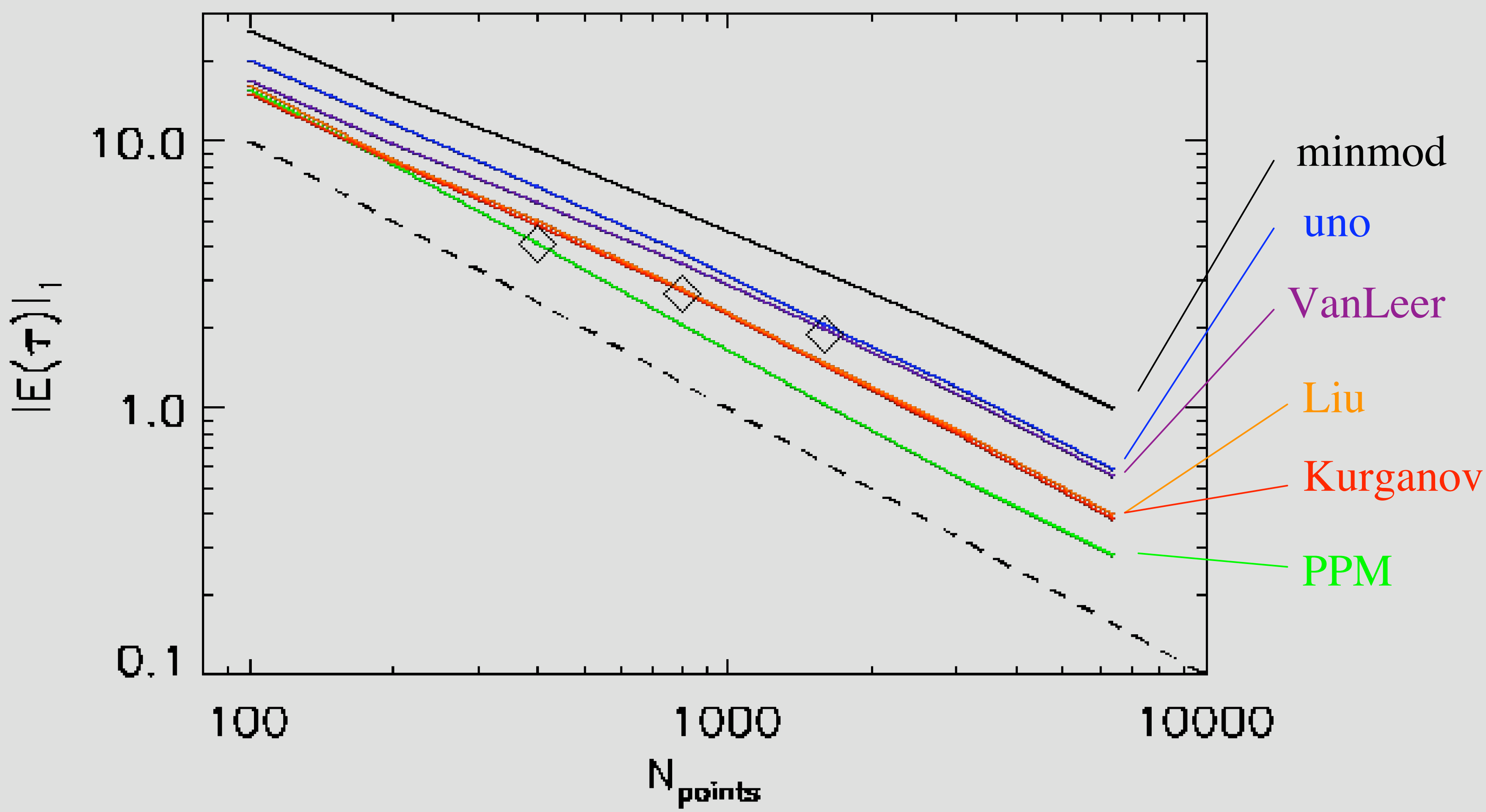
Ideal gas with $\Gamma = 5/3$
Left state: $\rho = 1$, $P = e3$, $v = 0$
Right state: $\rho = 1$, $P = e\text{-}2$, $v = 0$

Flow with Lorentz factor of 3.59



Solution with PPM reconstruction and 1,600 points at t = 0.36

Special Relativistic Test Problems: Stronger Shock



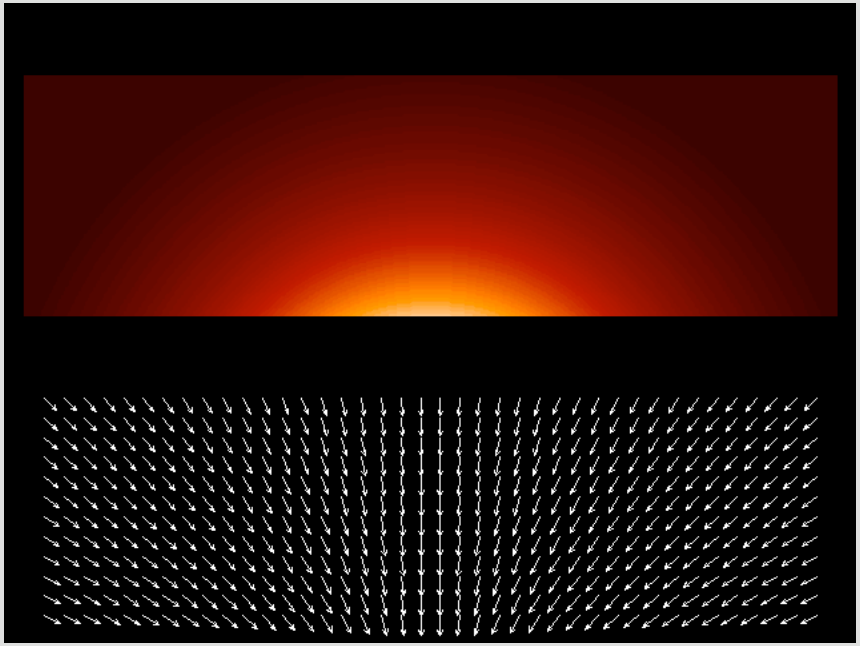
Diamonds are from Marti & Müller (1996) using the exact Riemann solver.

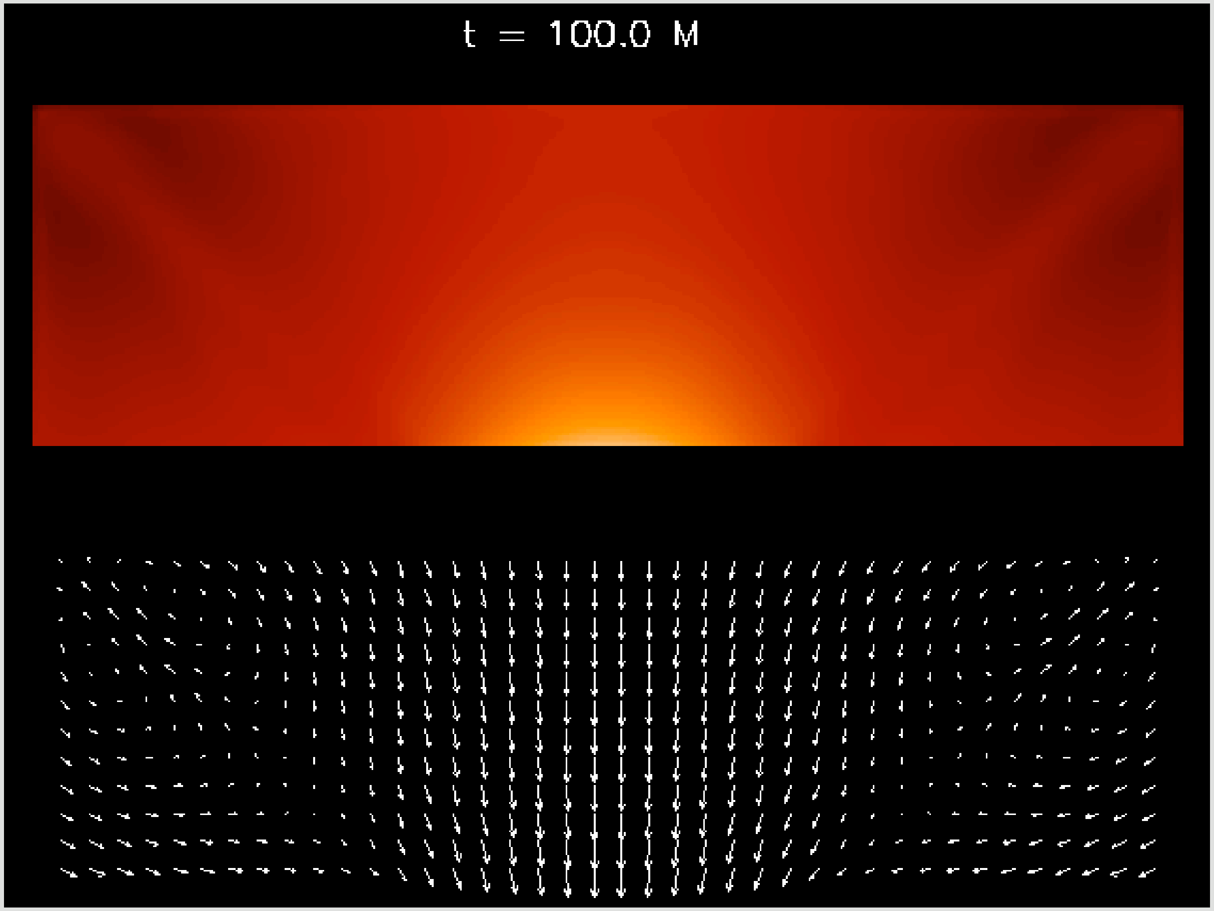
General Relativistic Test Problems: Bondi Inflow

Test of the geometric source terms for energy and momentum. Using isotropic coordinates,

$$ds^2 = - ((1-M/2r_{iso})/(1+M/2r_{iso}))^2\, dt^2 + (1 + M/2r_{iso})^4\, (dx^2 + dy^2 + dz^2)$$

Impose analytic inflow solution on 5 boundary faces of a grid spanning -10M < x,y < 10M and 4M < z < 10M and set outflow boundary conditions on the face closest to the black hole




t = 100.0 M

General Relativistic Test Problems: Coupled Geometry and Hydrodynamics for a Spherical Shell of Matter

Geometry:

- Constraint evolution
- Geometry is solved at every R-K step of the fluid
- Regularization at $r = 0$ is imposed directly
- Isotropic coordinates in Maximal slicing gauge ($Tr(K) = 0$)

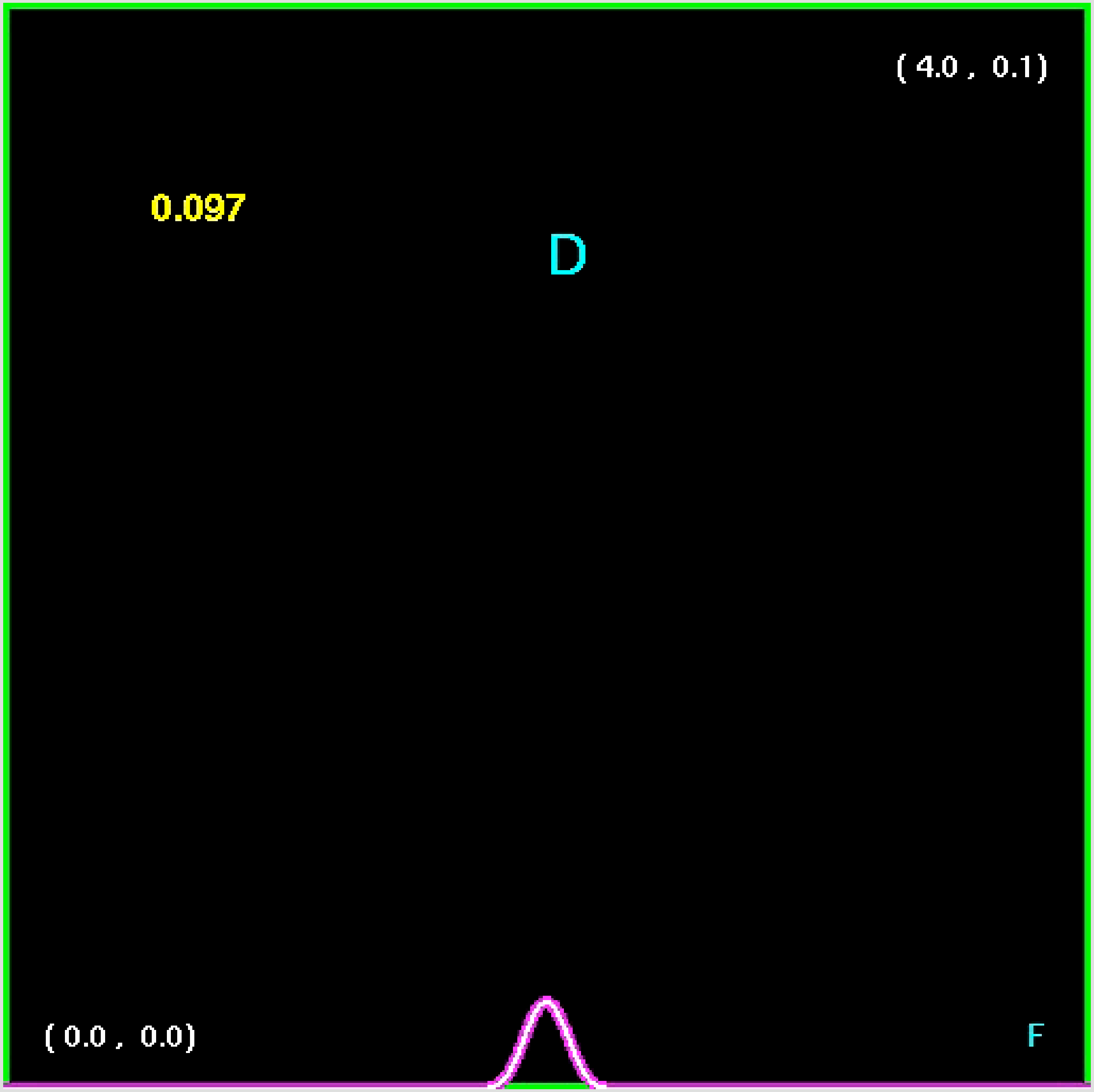
Hydrodyanmics:

- Valencia formalism
- Semi-densitized conservative variables
- Finite Volume scheme
- Test following implementations:

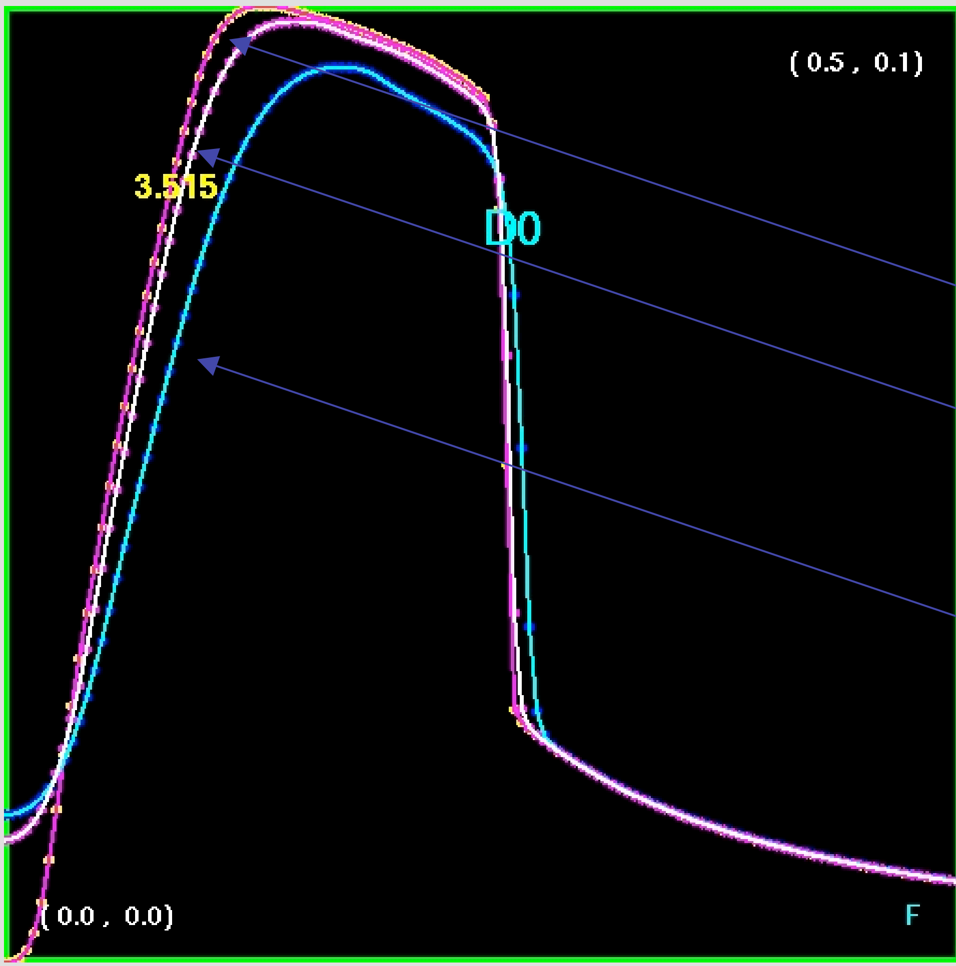
➢ Kurganov-Tadmor with maximum eigenvalue estimate

➢ Kurganov-Tadmor with $a = c = 1$

➢ Roe scheme


(4.0 , 0.1)
0.097
D
(0.0 , 0.0)
F

General Relativistic Test Problems: Coupled Geometry and Hydrodynamics for a Spherical Shell of Matter



Roe scheme

Kurganov-Tadmor with characteristic speed

Kurganov-Tadmor, a = 1

Conclusions

An overview and some results from model problems for the relativistic hydrodynamics code Flow-er

In the relativistic limit, it is possible to obtain numerical solutions of comparable quality to other high resolution, shock capturing schemes but without solving Riemann problems or even knowing any characteristic information from the flow.