AMR, Nonlinear Wave Equations and Relativity

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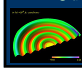
Modeling Gravitational Collapse:

- Difficulties:
  - Coordinates
  - Stability (form of equations)
  - Stability (approximation scheme)
  - Singularity formation
  - Boundary treatment
  - Range of physical scales
  - Computing resources
- Previous work in 1D and 2d:
  - 1D spherical symmetry
  - 2D axisymmetry

AMR can directly help



Choptuik,Hirschmann,Liebling,Pretorius, PRD 68,044007(2003)gr-qc/0305003
Choptuik,Hirschmann,Liebling,Pretorius, CQG 20, 1857 (2003)gr-qc/0301006

Lessons Learned:

- Tendency for "good" results to appear only above some minimum resolution
- Visualization and control over data important
- Small turn around time needed

It's hard!

The Move to 3D GR

- Get a handle on computing resources and visualization first… toy model in 3D
- Lots to explore:
  - Singularity excision within horizon
  - Different formulations & differencing schemes
  - Interesting matter sources:
    - Fluids
    - E&M
    - MHD

The Nonlinear Sigma Model

- Map from Minkowski (3+1) to target manifold (S^3)
- Symmetry breaking SO(4) → SO(3)
- Map satisfies: $\partial^\mu \partial_\mu \phi^A + \Gamma^A_{BC} \partial_\mu \phi^B \partial^\mu \phi^C = 0$
- NO gravity
- Choose hedgehog ansatz (spherical equivariant):

$$\phi^a = \begin{pmatrix} \sin\chi(r,t)\sin\theta\sin\varphi \\ \sin\chi(r,t)\sin\theta\cos\varphi \\ \sin\chi(r,t)\cos\theta \\ \cos\chi(r,t) \end{pmatrix} \qquad \ddot{\chi} - \frac{1}{r^2}\left(r^2\chi'\right)' = -\frac{\sin(2\chi)}{r^2}$$

- Already Known:

small initial data – no singularities "global existence"
large initial data – forms singularity

What happens between small and large?

- Parameterize initial data: $\chi(r,0) = Ae^{-(r-R_0)^2/\delta^2}$, $\dot{\chi}(r,0) = 0$
- Find:
  - A_{high} for which singularity forms
  - A_{low} for which energy disperses
- Tune for threshold A^*
  - Singularity formation for: $A > A^*$
  - Dispersal for: $A < A^*$

Critical Behavior

Liebling,Hirschmann,Isenberg, JMP 41,5691(2000)
Bizon,Chmaj,Tabor, Nonl.13,1411(2000)

- Evolution approaches a self similar solution
- Critical solution is one of a family of solutions found by Aminneborg,Bergstrom, PLB 362,39(1995)astro-ph/9511064
- Critical solution sits on codimension 1 boundary between two basins of attraction
- Single unstable mode takes solution away from this boundary manifold

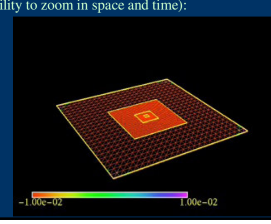
Move into 3 Spatial Dimensions

- Map satisfies: $\partial^\mu \partial_\mu \phi^A + \Gamma^A_{BC} \partial_\mu \phi^B \partial^\mu \phi^C = 0$
- 3D … pick Cartesian coordinates
- Pick generalized hedgehog: $\chi(r,t) \to \chi(x,y,z,t)$
- Nonlinear equation: $\ddot{\chi} = \chi_{,xx} + \chi_{,yy} + \chi_{,zz} - \frac{\sin 2\chi}{r^2}$
- Initial data: distorted Gaussian pulses, toroid, etc
- 3D is harder:
  - Tendency toward more fields
  - More memory/field 1000pts->8kB vs (1000)^3->1GB
  - More operations per point

need more computing power

Resolving power

- Importance of visualization (ability to zoom in space and time):



Distributed AMR

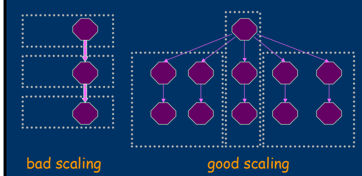
- Berger & Oliger type AMR:
  - Even integer refinement ratio
  - Restricted to single parent
- Typical near-critical:
  - 15-20 levels
  - 2:1 refinement

- AMR spread across many processors
- Strategy: distribute entire grids to different proc's
- Scheme:
  - master process orchestrates
  - slaves loop for commands
- Pro:
  - Straightforward to implement
  - Works well for large numbers of grids per proc
- Con:
  - Not optimal, would like more autonomy
  - Horrible scaling for completely nested grids

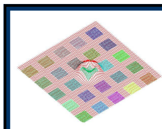
Looks like serial code

Distributed AMR: Two Regimes

- Vertical
- Horizontal

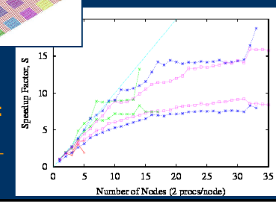


Early Distributed AMR Scaling



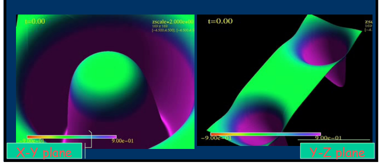
Speedup for N processors:

$$S = \frac{T_1}{T_N}$$



Flatspace Critical Phenomena

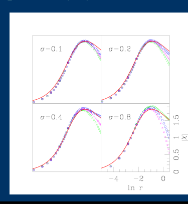
- Tune based on singularity formation
- Self-similar critical solution appears
- Toroidal Initial data



Results in 3D

Liebling, PRD 66,041703(2002) gr-qc/0202093

- Same, spherically symmetric critical solution



Other Models

…work in progress

Other Models: Semilinear Wave Eq.

Bizon,Chmaj,Tabor, math-ph/0311019

- Simplest nonlinear wave equations
- Cartesian coordinates

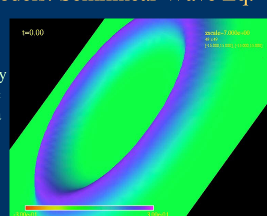
$$\ddot{\chi} = \chi_{,xx} + \chi_{,yy} + \chi_{,zz} + \chi^p$$

- Spherically symmetric results show "conventional" critical behavior for p=7

$$\chi^*(r,t) = (T-t)^{-1/3} U_1\left(\frac{r}{T-t}\right)$$

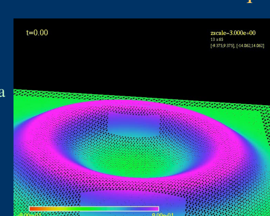
Other Models: Semilinear Wave Eq.

- p=7
- Spherically symmetric initial data



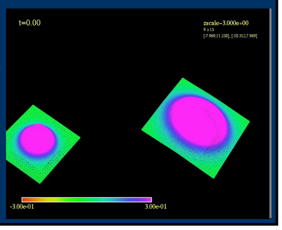
Other Models: Semilinear Wave Eq.

- p=7
- Toroidal initial data



Other Models: Semilinear Wave Eq.

- p=7
- Two initial pulses



Other Models: Semilinear Wave Eq.

- p=7: spher. symm., self-similar solution
- p=5: static solution serves as critical solution
- p=3: late-term behavior messes up crit. search

- p=7: spher. symm. solution picks up no more unstable modes
- p=5: static solution no longer attractor, but still on threshold; not clear what critical solution is
- p=3: similar late term solution

Other Models: Maxwell Dilaton

Hirschmann, Liebling

- Action

$$S = \int d^4x \sqrt{-g}\left[R - a_0(\partial\phi)^2 - a_1 e^{-2\kappa\phi} F_{ab}F^{ab}\right]$$

- Choose flat space (R=0)
- Dilaton attractive… possible critical behavior
- Can't have spherically symmetric solutions
- Results not clear yet….looks like critical behavior though

Ideal Distributed AMR Framework?

- Well-documented with readable code
- Easy to compile w/ simple dependencies
- Well-thought out, conceived, and implemented…extensible
- Easy to define your fields and incorporate EOM
- General…cell centered/vertex centered
- Good distributed scaling
- Can checkpoint
- Nice output options

"Grid" based computing

- Commonality of uses:
  - Domain decomposition
  - Adaptive Mesh Refinement
  - "overlapping" grids ala (Neilsen, et al), (Scheel, et al)
  - Patches on sphere
  - Possibly abutting grids
- Commonality of functionality:
  - Create/destroy grids on a specific processor
  - Advance/step grids in time
  - Sync grids with siblings/parents/children
  - Output fields defined on grid

- Commonality of properties
  - Collection of fields
  - Coordinates
  - Relationship to other grids
  - Etc (e.g. time to which advanced, owning process)

Creation of a general grid-based library would leverage/pool resources

Long Term Goal: General Code

- Fully **gravitating**
- **3D** – no assumed spatial symmetry
- Coupled to **Magnetohydrodynamics (MHD)**
- Good general coordinates
- Resolved (far-field: waves; near-field: holes): **AMR**
- Robust – stable for strong field and matter sources
- Distributed for supercomputers/clusters – **MPI**
- Deals with singularities – **excision**