Waveforms, simulations and future prospects?

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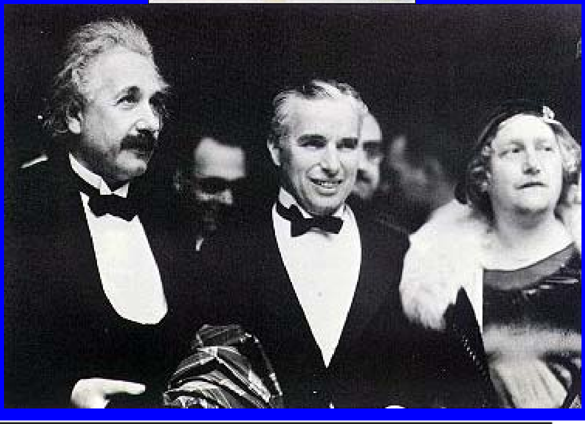
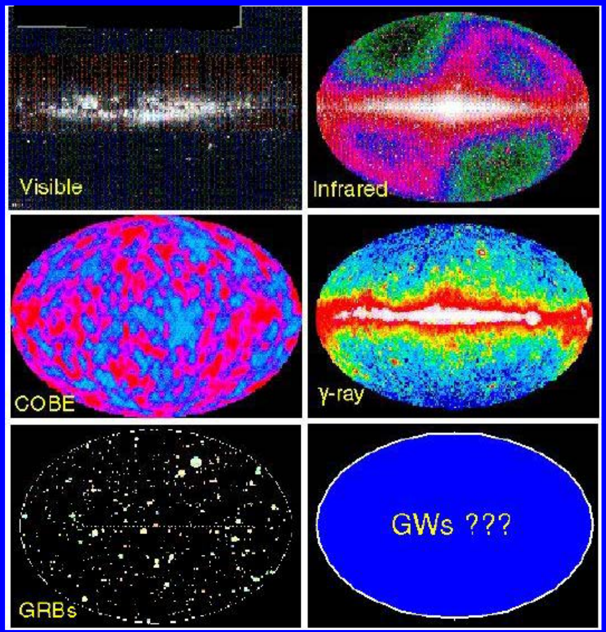
Outline

- Overview of the problem & num. rel. goals
- Recent results
- Implications
- Caveats
- Next steps

Gravitational Wave astronomy…

- Gravitational waves predicted by Einstein’s theory.
  - Produced by coherent motions of masses/energy
  - Propagating at the speed of light
  - Mostly unscattered from source to receiver
  - Frequencies complementary to the EM spectrum
  - Observables in GW ~ $1/r$; in EM ~ $1/r^2$





LIGO

What do we know? (when do we know?…)

- Linearized theory
  - Post Newtonian expansion ($v \rightarrow 0$, $M/D \rightarrow 0$) reasonable good handling to some given orders. *[at least enough for $200 GPSs….]*
  - Perturbations over fixed backgrounds. Good handling to 1st order in special cases, iffy from there on….

- Non-linear theory
  - Global stability of flat spacetime understood in 1990 [Christodoulou-Klainerman, also Lindblad-Rodnianski 05].

- We'd like to know
  - Behavior around highly dynamical, strongly gravitating cases ($v \rightarrow c$, $M/D \sim 1$).
  - Behavior close to singularities, connection to quantum gravity ideas
  - Role in astrophysical phenomena.

Where to look for signatures of strong gravity?

- Astrophysical systems containing BHs, Neutron stars.
- Cosmological consequences of early universe scenarios
- Highly energetic configurations in the lab (assuming some flavors of string theory is correct).

All require understanding the solution of Einstein equations.

- *Non-linear, highly involved, constrained, PDE system.*
- *Many different length scales involved.*
- *Physics (2 d.o.f) 'hidden' in many variables.*
- *Singularities mark the demise of the theory, and we want to look 'close' to them.*

Numerical simulations only road to make head-ways.

- To obtain particular solutions
- To gain insights which can be exploited at the analytical level.

Astro-frontier

- Black holes
  - end point of sufficiently massive stars collapsing
  - end product of collisions of neutron stars

- How do we see them?
  - Effects on neighboring matter → EM, neutrino radiation. Eg Gamma ray bursts
    - Though EM can be scattered, direction dependent
  - Effects on the fabric of spacetime → Gravitational Waves
    - Even with strong sources (collisions) GWs are quite weak, detectors alone have a hard time to catch the waves
    - Even when ‘caught’, we need to interpret signals and extract physical information.

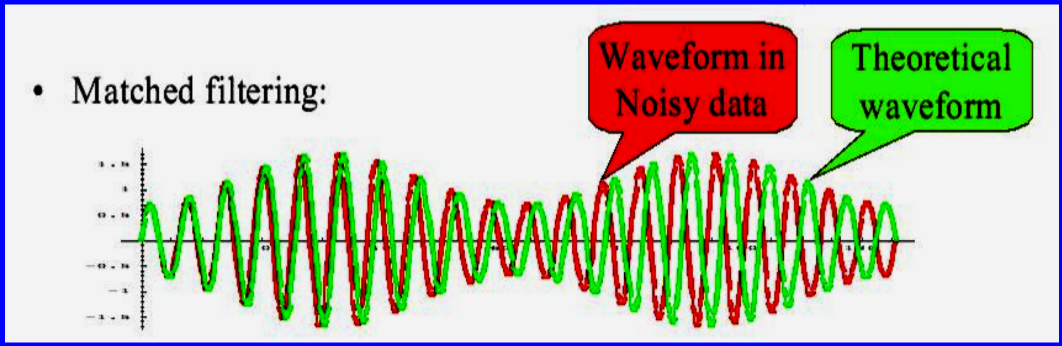
Mining out the physics… Matched filtering…

- Data analysis requires a large number of ‘templates’

• n: detector noise; T: template; h: signal

•$T := T(p_1, p_2, p_3, p_4 \ldots..)$; $\langle , \rangle$: inner product (time integration)

$S := \langle h|T\rangle / \langle T|n\rangle$



  – e.g. Binary Neutron stars <u>without</u> including spins ~ 10^5 analytical (easy parameterized) templates!
  – How to turn the results effectively and practically to the DA efforts?

(OLD?) challenges…….

- Obtain accurate waveforms from theoretical models.
  - Estimated radiation ~ a few % over several dynamical times
    - Error must be well below this
    - Cost estimates using a ‘straightforward’ implementation → a few days running on Petaflop machines (per case!)
    - Must use: higher order + grid adaptivity.
    - Can’t watch if things ‘look right’.

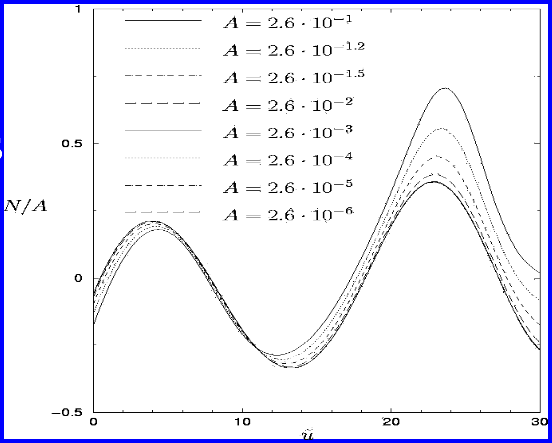
- Many templates required, parameter space is huge (e.g. 10^5 for binary neutron stars).

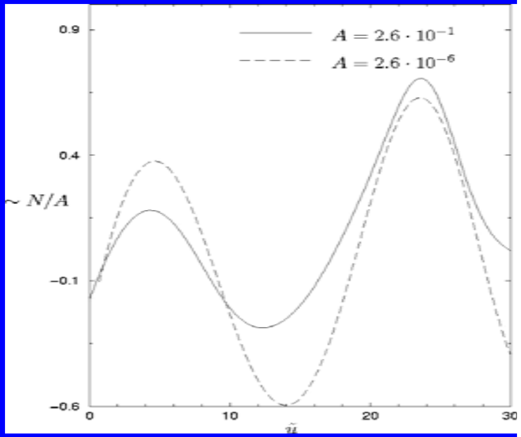
- Ultimately, a ‘network’ of efforts will have to be set back-and-forth from analysis to/from simulations.

What's in 'accurate' model

- Einstein eqns.
  - 10 2nd order PDEs. Coordinate freedom can 'knock down' 4, remaining eqns have 4 *constraints*.
  - Can be cast in symm hyperbolic form.
  - Relatively few rigorous results to use as guidance:
    - Well posed of IVP understood in the 50's, well posed of the IBVP just a few years ago (Friedrich-Nagy).
  - 'Painful' observations
    - Constraints seen to depart from constraint surface (a la $\nabla B = 0$ in MHD).
    - Too many, a priori equally good, formulation of Einstein eqns.

- Surprises/unexpected behavior can arise easily with strong implications for GW search and analysis.

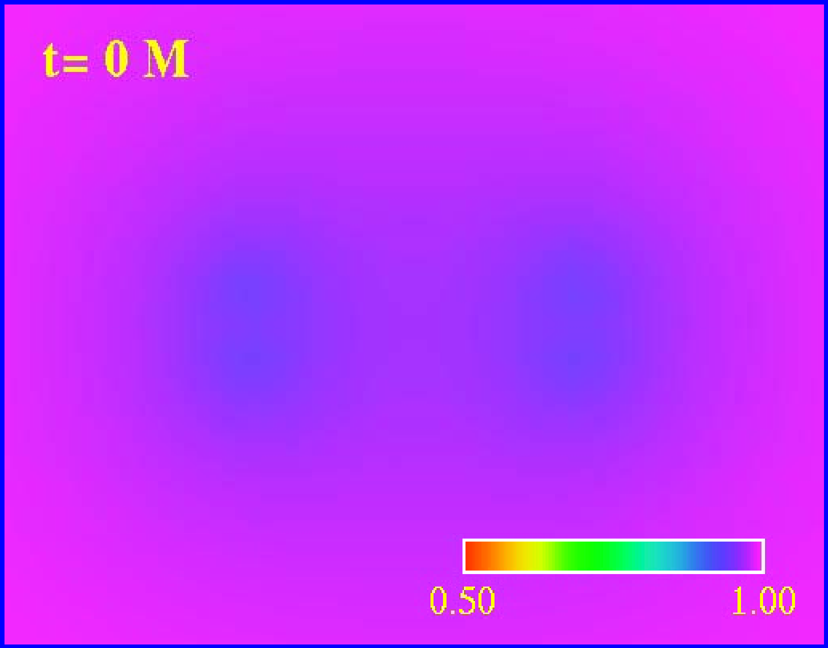


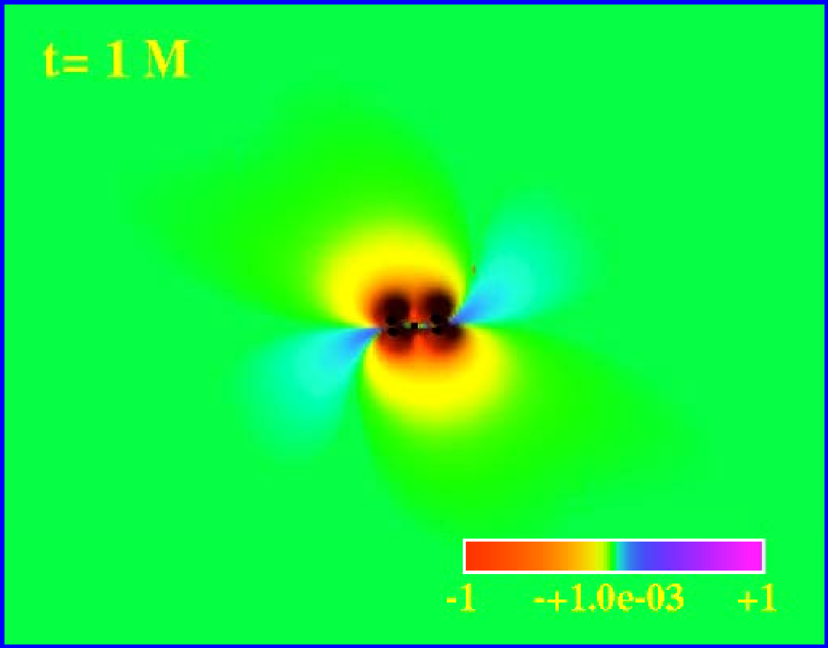


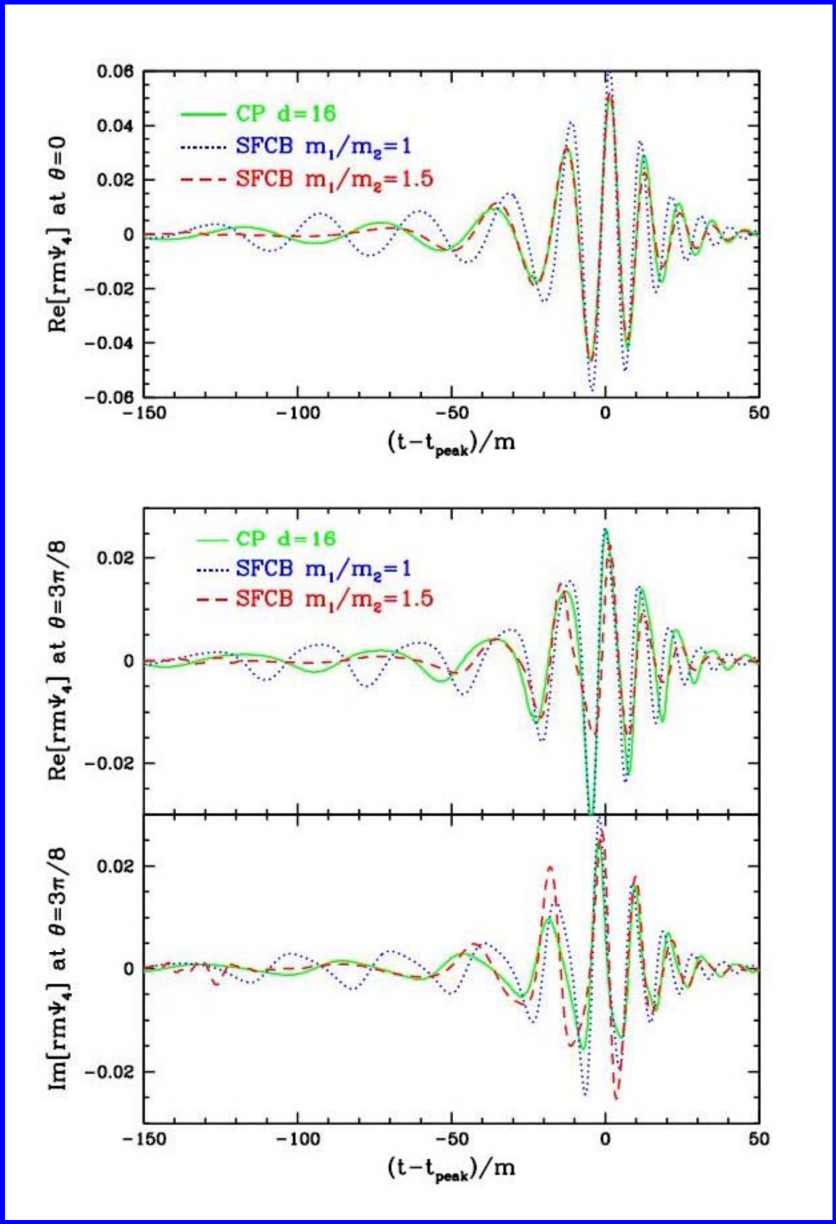
[Zlochower,LL,Winicour,Husa,Gomez, 03]

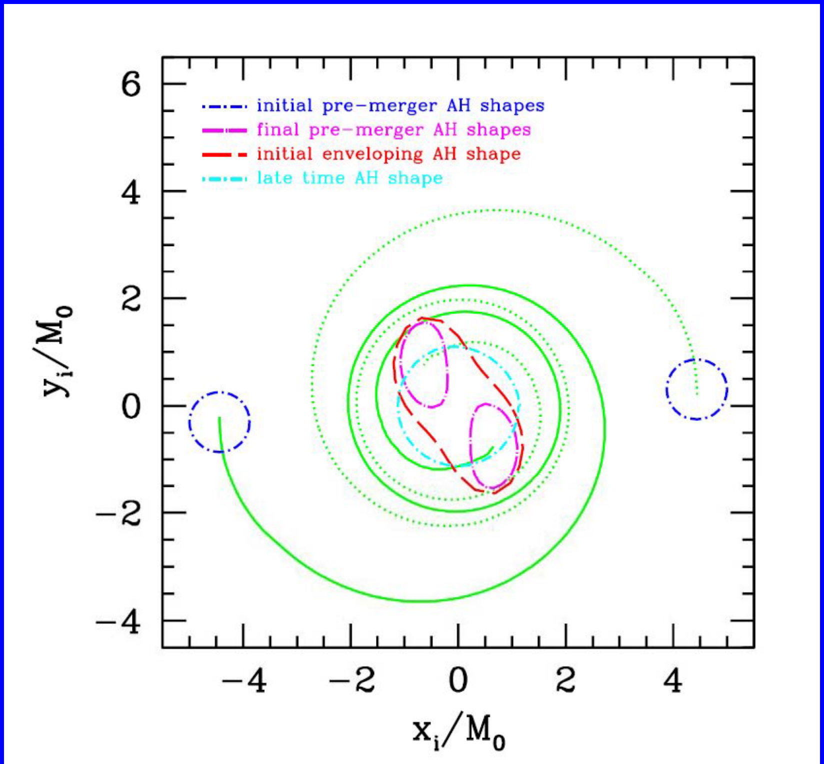
But…times have changed (?)

- Simulations of orbits have been obtained.
  - Pretorius [Harmonic formulation, excision of bhs, 3D]
  - Campanelli-Lousto-Zlochower (UTB) & Baker-Centrella-Choi-Koppitz-VanMeter (NASA-Goddard) [BSSN, no excision, *3-1/2D*]
  - Herrman-Laguna-Shoemaker (PSU); Diener-Takahashi-Pollney…(LSU-AEI); Bruegmann-Hannan-Husa-Sperhake (Jenna) ['reproduced' above]






CP d=16
SFCB m1/m2=1
SFCB m1/m2=1.5
Re[rmΨ4] at θ=0
(t−tpeak)/m
Re[rmΨ4] at θ=3π/8
Im[rmΨ4] at θ=3π/8


initial pre−merger AH shapes
final pre−merger AH shapes
initial enveloping AH shape
late time AH shape
yi/M0
xi/M0

Pretorius

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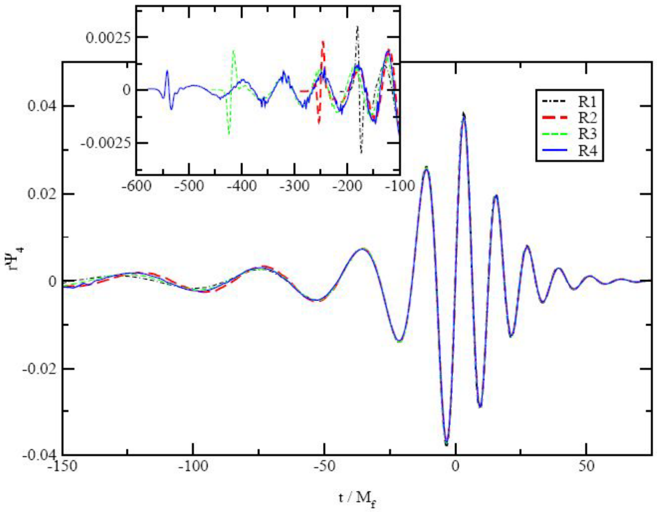


FIG. 5: Waveforms from runs R1 - R4. The figure shows nearly perfect agreement after $t = -50M_f$. For the preceding $500M_f$, shown in an inset, the waveforms agree in phase and amplitude within about 10% except for a brief initial pulse at the beginning of each run.

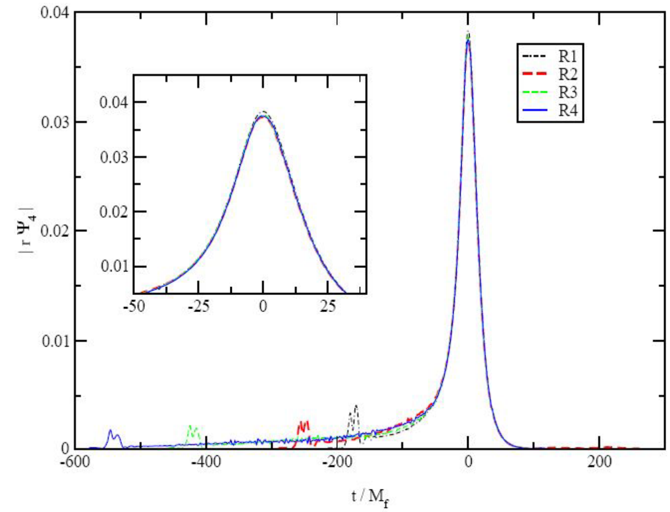
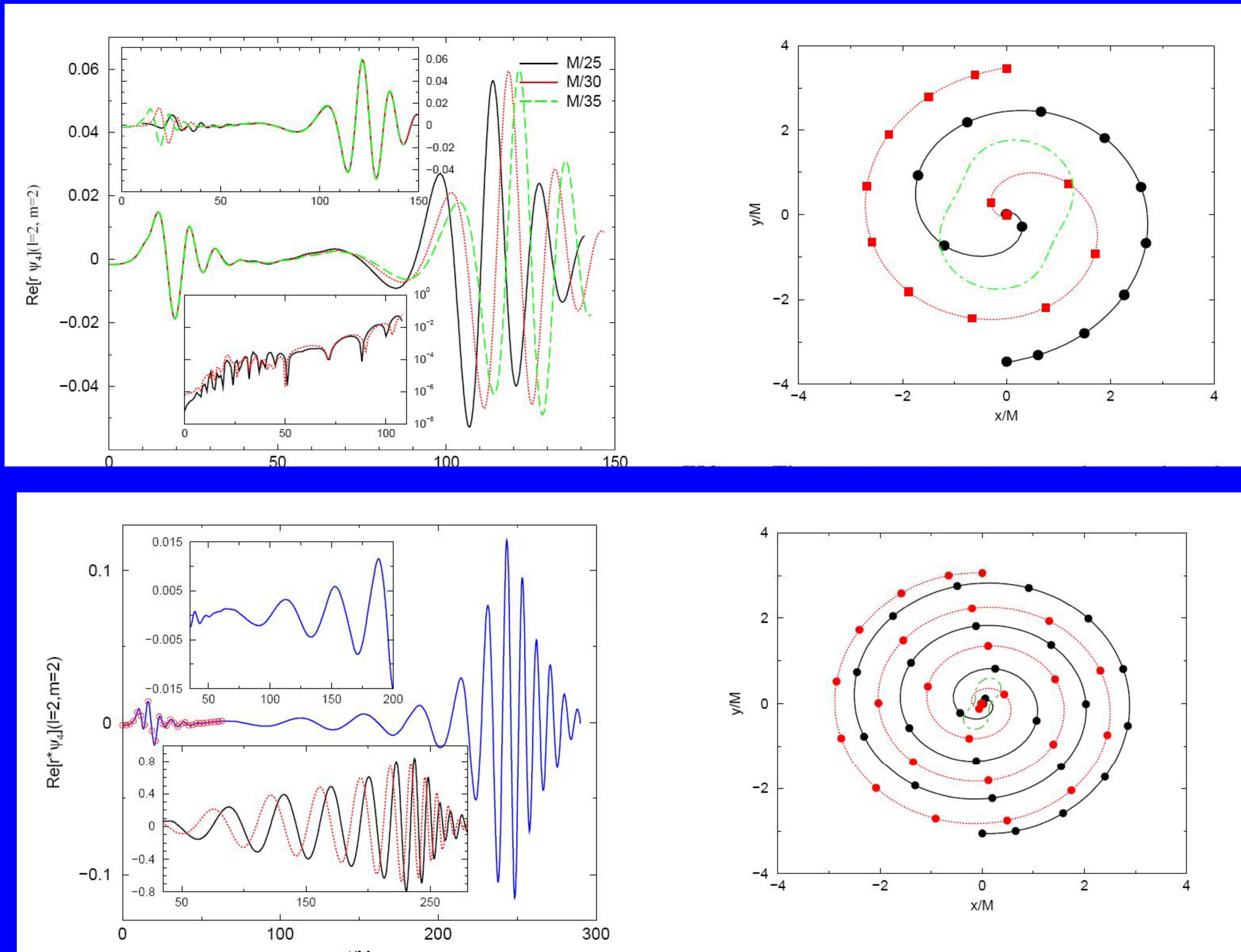


FIG. 6: Amplitudes, absolute value of (complex) Ψ_4, of the waves. The curves have been shifted such that the maxima are all at time 0. The inset zooms into the peak showing the strong agreement from $t = -50M_f$ on. We have used the amplitude peak as a reference to align our simulations in time.

Baker-Centrella-Choi-VanMetter-Koppitz



Campanelli-Lousto-Zlochower

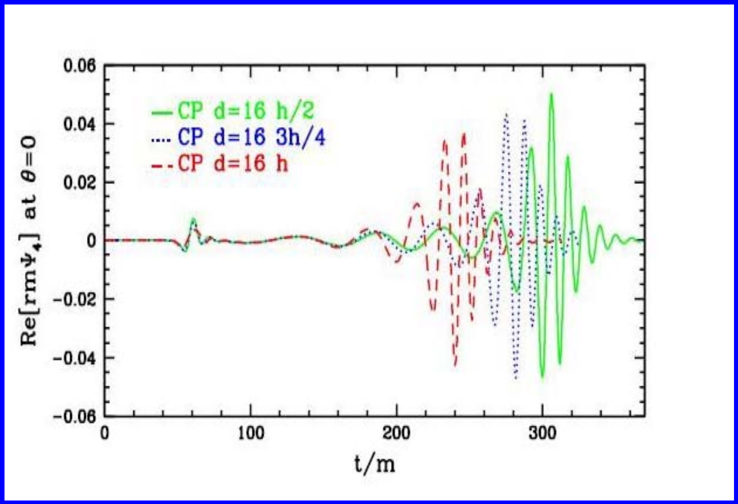
Conclusions drawn

- Radiation ~ 5% of total mass. In the ball park though stronger than anticipated.
- No appreciable non-linear effects. Quadrupole formula pretty good.
- No “ISCO”, ie. No marked transition from inspiral to plunge.
- Kicks (recoils) within the handwaving predictions.

Are we surprised?

Caveats to keep in mind

- Agreement in resolutions require shifting in time.
- Convergence studies not sufficiently convincing (e.g. d = M/20, M/24, M/28 isn't really varying things by much).
- Waveforms/kicks calculated through Newman Penrose ψ_4 aren't checking crucial issues are satisfied.



ψ_4 is a combination of the Weyl tensor coms: $\psi_4 = C_{abcd}\, n^a\, m^{b*}\, n^c\, m^{d*}$

But:

- Caveats due to systematic and conceptual problems must be looked into

Caveats….

- Waves and kicks
  - Tetrad (l,n,m,m*) is defined at future null infinity
  - Induced angular metric g = S + C/r with S *the unit sphere metric*
  - $\psi_4 = \sigma^*_{,uu}$
  - $g_{uu} = 1$; $g_{uA} = 0$ (inertial observers stay at const angles; clocks tick the same)
- Resolution issues
  - Over the whole evolution ~5% radiated ➔ error per step better be well below 5%/number of steps. Analytical estimates show, error per wavelength behave as:

ε	2nd	4th	6th
10^{-2}	26	8	6
10^{-3}	81	15	9
10^{-4}	257	27	13
10^{-5}	816	48	19

Nonlinearities likely to be
Induced by size of holes ~ M
Resolutions being employed
D ~ M/10
# of steps ~10^4

More caveats

- Initial data so far restricted in models and physical situations

- BSSN evolutions doing something 'funny' with the singularities, will they behave the same as resolution is significantly improved? Nevertheless, results similar to the excision ones.

- Learn from our neighbors? Astrophysicists have been evolving binary stars in Newtonian theory for a while….
  - Without using variables adapted to the problem + methods that conserve to round-off level crucial quantities… binaries merged within a few orbits! (Swesty 90's!). Use as one of the main variables the angular momentum + coordinates adapted to the symmetry + refined methods, orbits maintained over several dozens revolutions (Tohline etal late 90s).
  - Could this be affecting the outcome?

- We'll come back to caveats later… for now, mostly ignore them and press ahead… what can we say about the waves for data analysis?

Data analysis connections

- Work with Brady,Pretorius,Baumgarte,Creighton,Devoe.
- Qns:
  - With the current info, can we say anything about :
    - ‘reaction’ of data analysis to these waveforms?
    - Parameterization of waves to use in data analysis?
    - Are errors in the waveforms ‘visible’ by the detectors (I.e. are 2 ‘equal’ waveforms at 2 different resolutions the same for data analysis?)
    - Can one begin to draw an idea on how many templates one must have?

Equality of waveforms

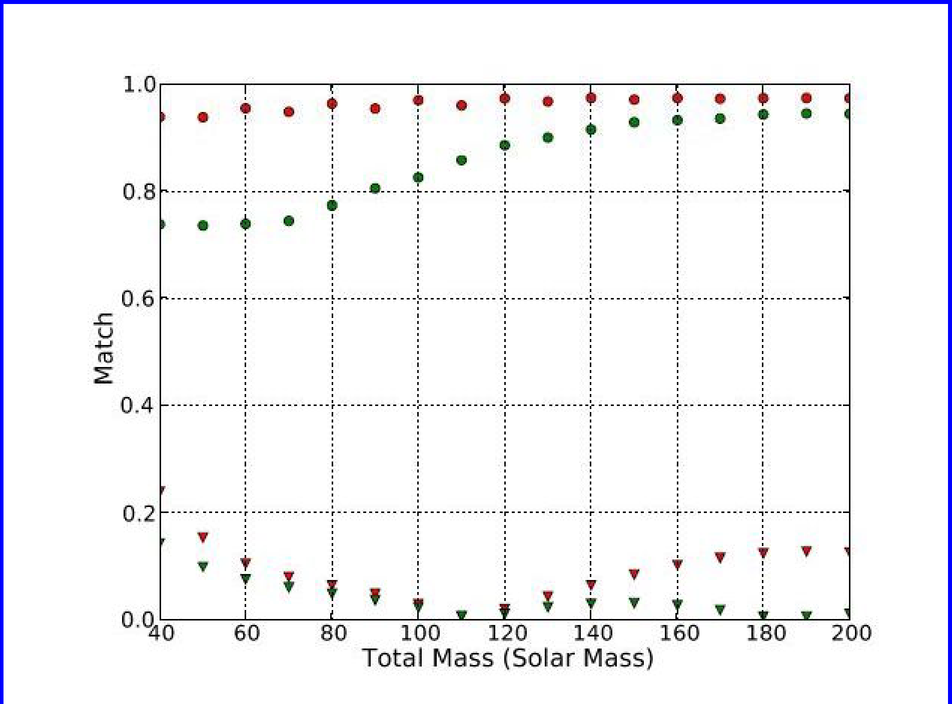
- ‘same’ waveforms, at different resolutions. Compute differences ‘weighted’ by the detector. I.e. differences per frequency and vs detector noise.

For BBH, masses can be re-scaled.

Compute $<h_1(t),h_2(t-t_o)>$ and vary over all t_o

Look for when the match is

highest.



Pessimistic view: match is rather poor!

Optimistic view: shifted match is really good!

- Shift in time needed vs mass.
- linear dependence! → difference in waves captured by mass rescale?
- Indeed, if searching 'blindly' good matches found with masses higher than the ones known from simulation.

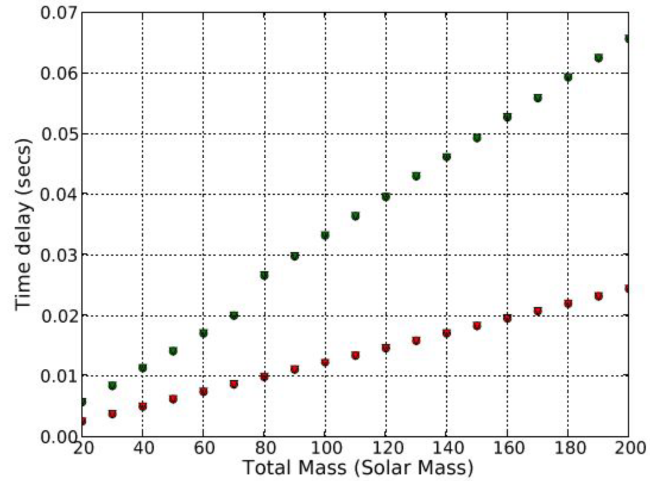


FIG. 5: The time-offset which maximizes the match (for the same mass) between the h_+ waveform polarization computed as the overlap between the waveforms extracted from two different resolution runs. The green uses the finest simulation as the waveform and the coarsest simulation as the template; the red uses the finest simulation as the waveform and the next finest simulation as the template. While this shows con-

Parameterization employed:

$$w(t,\lambda) \sim A\,(1Mpc/D)\,[\cos(\Phi)\,e_+(t-t_0,m_1,m_2) + \sin(\Phi)\,e_x(t-t_0,m_1,m_2)]$$

$$A,\Phi \sim F(ascension, declination, polarization, inclination, time)$$

While it'd seem the finest resolution could be good for data analysis:
- issues in shifts→ masses point to difficulties in tying sources with signals
- systematic effects need to be addressed to be sure and remove/reduce this problem

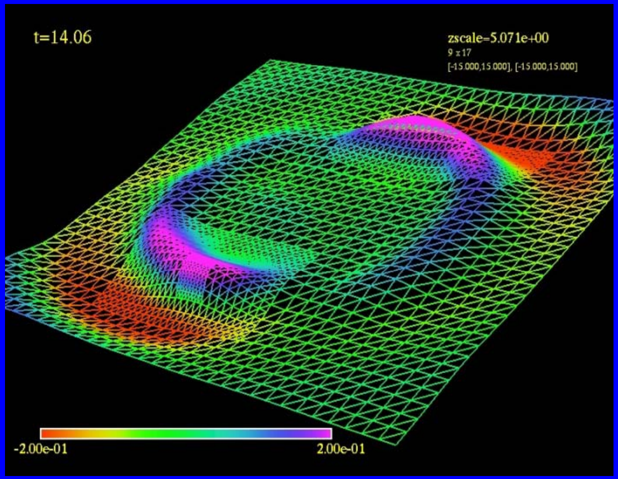
addressing caveats

- Waves: Short of reaching future null infinity, some issues can/must be looked into ([LL-Moreschi]):
  - Suitable fall-off must be satisfied, dependence on extraction location examined and making sure outer boundaries are not influencing results. ✓✓✓?
  - Induced angular metric at the extraction worldtube is conformal to the unit sphere metric. $G(t,\theta,\phi) = F(t, \theta,\phi)\, S(\theta,\phi) + O(1/R)$
  - $\Psi_4 = \sigma_{,tt} - D(F_{,t}/F) + \sigma\, (F_{,tt}/F) + 2\,(\sigma_t F_t/F - \sigma\, F_t^{\,2}/F^2)$
  - 'shifts' in observer's motion must be corrected for if non-zero g_{tA}
  - Tick-rates must be accounted for if g_{tt} not 1.

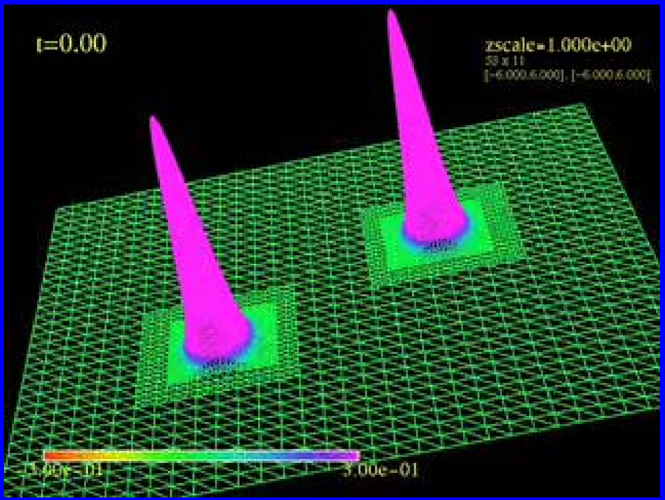
  - If not bothering to do this, at least a measure of these effects is necessary.
  - $\|R-2\| = 0$? ($R = 2\,[F^2 + D^A D_A \log F]$ otherwise & solve for it)
  - Estimates on this being carried out in Pretorius simulations, already indicating this *is* an issue. Will be assessing the influence in the near future.

Resolution & convergence issues

- Estimates indicate → AMR + higher order is needed.
  - AMR, add (discard) grids when (not) required.
  - In linear problems, grid-structure can be a-priori specified. In non-linear ones it isn't possible as unexpected features may arise.
  - Must have a way to check the solution with 'itself' and adjust grids accordingly (self-shadow hierarchy).



  - Current efforts giving waveforms
    - PAMR (Pretorius). True-AMR, though 2nd order
    - Carpet (Schnetter). Pre-determined grid structure, fourth order operators in place, though artificial boundaries can strongly affect the results & convergence rates.
    - HAD . True AMR & arbitrary orders.
    - Other packages out there! Why reinvent the wheel?

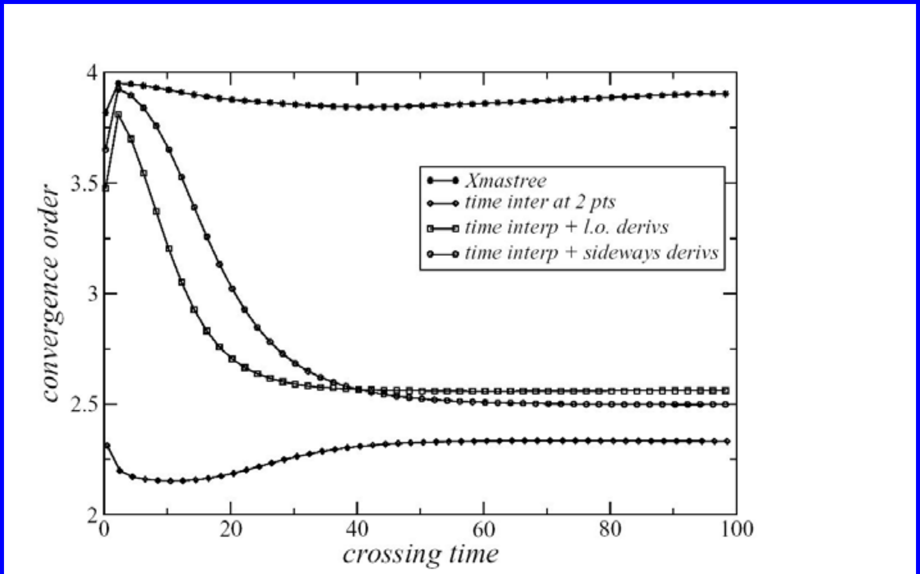


AMR & higher order issues

- Artificial boundaries are introduced.
- Integration 'requires' boundary conditions.
- Traditionally done by interpolation of 'parent' level → this is 2nd order accurate. Alternatives introduced bias and, worse yet, instabilities!

- Tapered (Xmastree) approach remove this issue (at a cost) but ensures convergence at the desired order. [LL,Liebling,Reula]

Δx $\Delta x/2$



Beyond convergence

- No matter what/how we do the simulations, cost will be high, typical simulations take really long time to finish. For better accuracy, Richardson extrapolation can be employed.

$$F(d) = F_A + d^2 g + d^p h$$

$$F(d/2) = F_A + (d/2)^2 g + (d/2)^p h$$

$$so,$$

$$F_A = 4F(d/2) - F(d) + O(d^p)$$

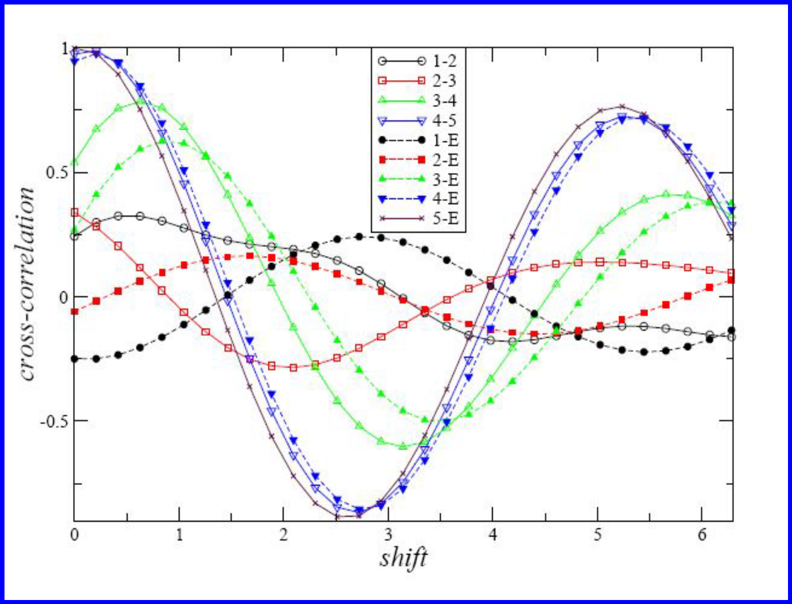
- So… what does this mean for us?
  - Take a crude mock-up of ‘chirping’ waves

$$F = t^2 \sin(w(t)t) \quad ; \quad w(t) = 1 + at^2$$

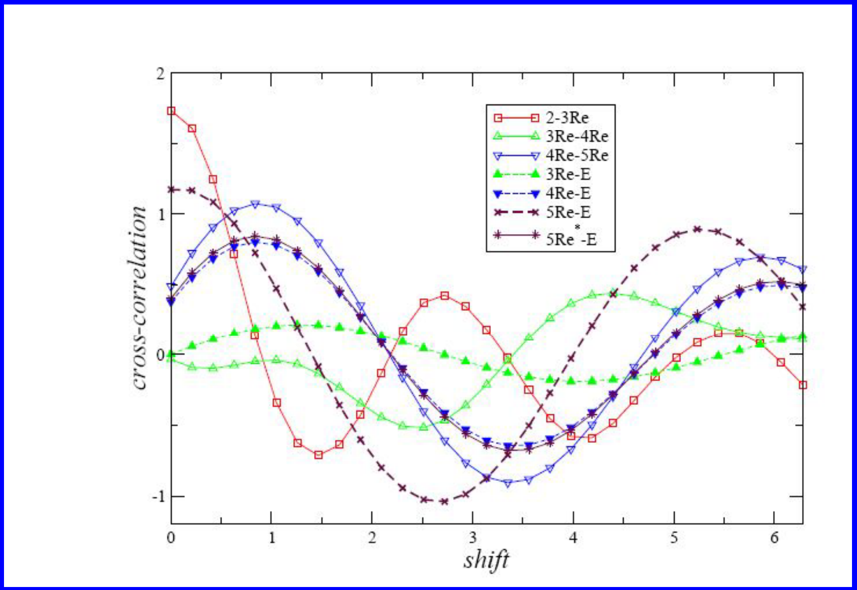
$$\hat{F}(d) = t^2 (1 + d^p) \sin(\hat{w}(t)t) \quad ; \quad \hat{w}(t) = w(t)(1 + d^p)$$

  - And compute cross-correlations among themselves, with the analytical answer and the Richardson extrapolated results…

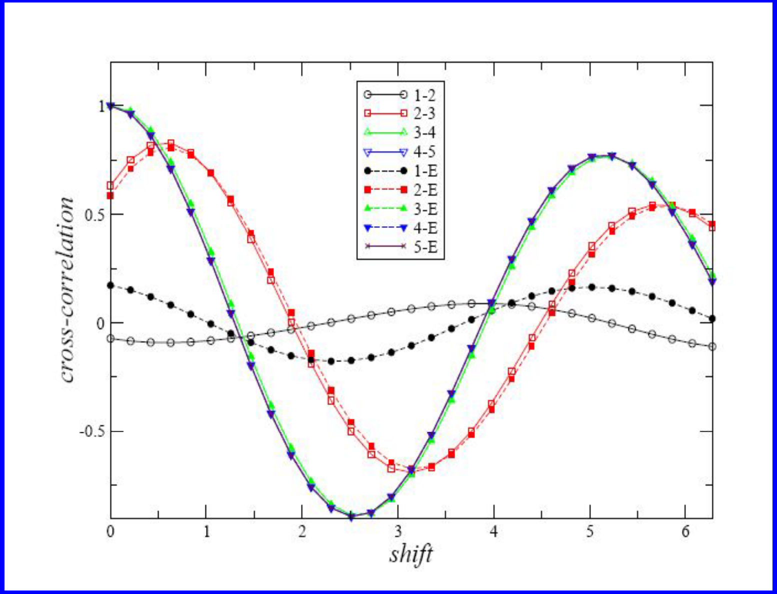
2nd



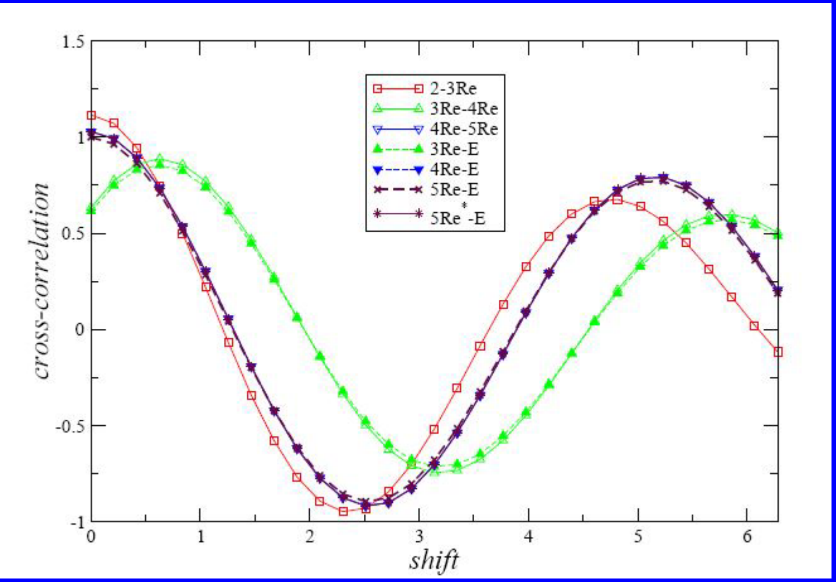
2nd Richardson



4th



4th Richardson



So what are the messages?

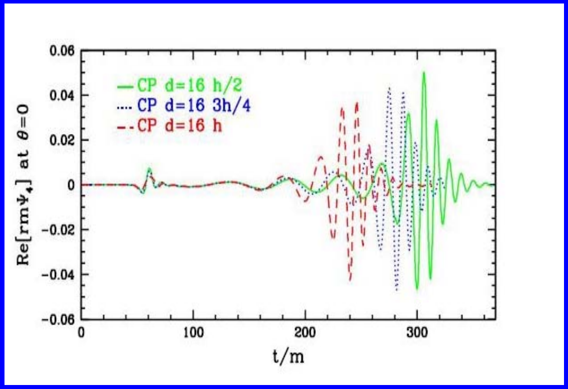
- The good:
  - Several simulations do get orbits, and there is good qualitative agreement. Even with reasonable different initial data.
    - This need not be surprising, what else is there?

- The bad?
  - Some possibly crucial issues have been ignored (waveform extraction)
  - Initial data restricted so far (but this is the nature of the field)
  - Still to understand what goes on on the puncture side of things.

- The ugly?
  - Isn't the picture too simple?
    - Waves mostly described by the quadropole formulae!
      - Templates will be much easier to generate and encode. (ugly is in the eye of the beholder).
    - Where did the non-linearities go? – will we be able to actually probe GR?
      - First non-trivial example of no surprises in GR when probing non-linear regime?
      - Where do we go to look for insteresting things? Matter models… BH-NS, NS-NS?

Testing/confirming observations…

- Orbits-merge driven by physics or by errors? (OK qualitatively but not quantitatively)



  - Must examine some cases with true AMR and guaranteed higher order convergence [Palenzuela,LL,Liebling]

- Does quadrople formula rule?
  - If true, should be valid for all binaries as long as objects remain compact. Binary NSs [Anderson,Olabarrieta,Motl,LL,Neilsen,Hirschmann] and other ‘exotic’ binaries being considered & [Palenzuela,Olabarrieta,LL,Liebling]

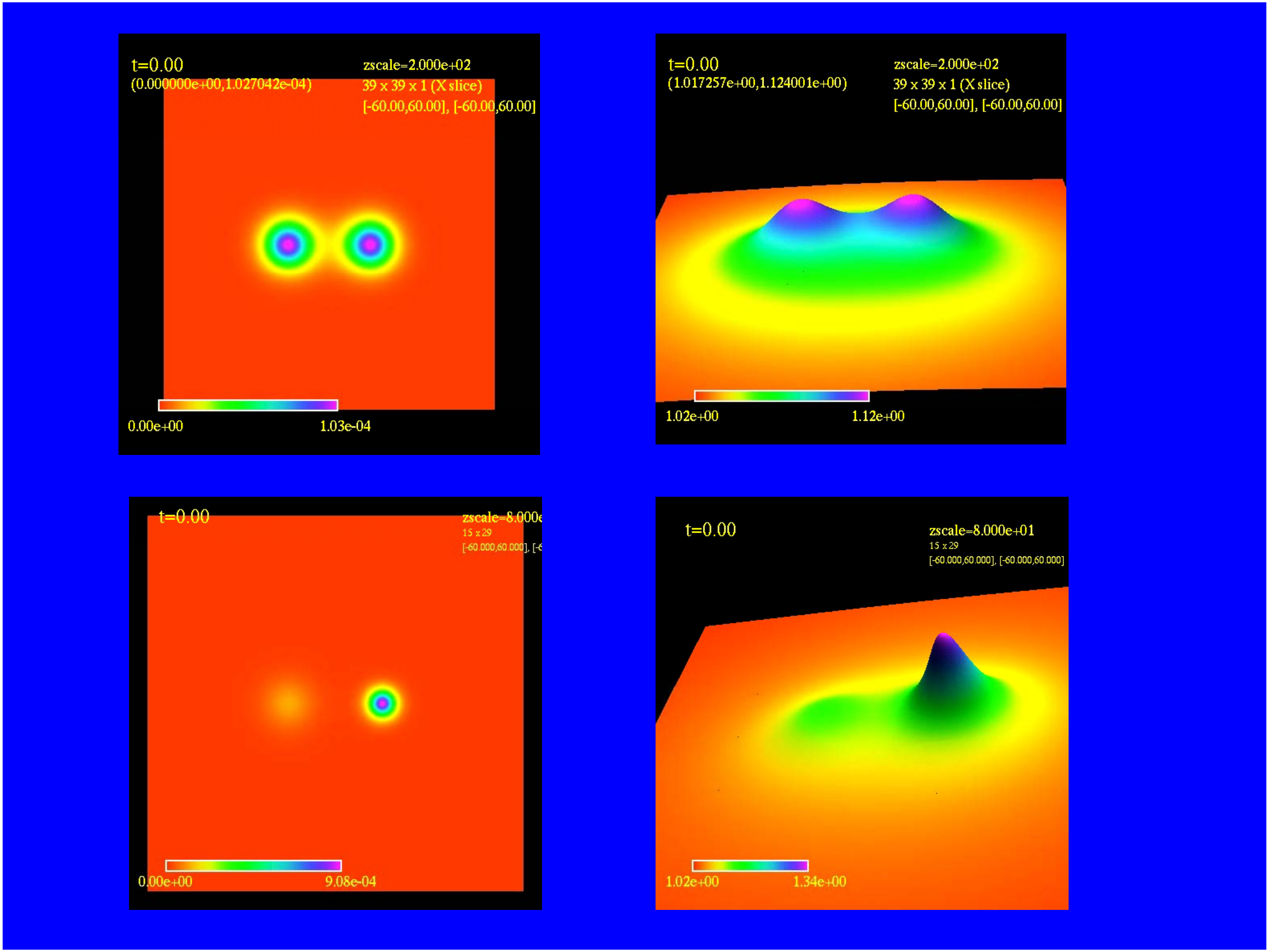
Exotic binaries, Boson Stars

- Boson stars. Compact objects from a complex scalar field
  [Kaup,Bonnazolla-Ruffini]

$$T_{ab} = (\nabla_{(a}\phi\nabla_{b)}\bar{\phi} - 1/2\, g_{ab}[g^{cd}\nabla_c\phi\nabla_d\bar{\phi} + m^2(|\phi|^2)/2])$$

- Resulting ‘stars’ share features with TOV stars.
  - Have both stable and unstable branches.
  - Stable stars remain coherent
  - Unstable stars either collapse to a BH or disperse away.
  - Yet… do not yield shocks or contact discontinuities, nor do they have singularities lurking in them.

- Initial data, superpose 2 boson stars, resolve constraints and evolve resulting configurations.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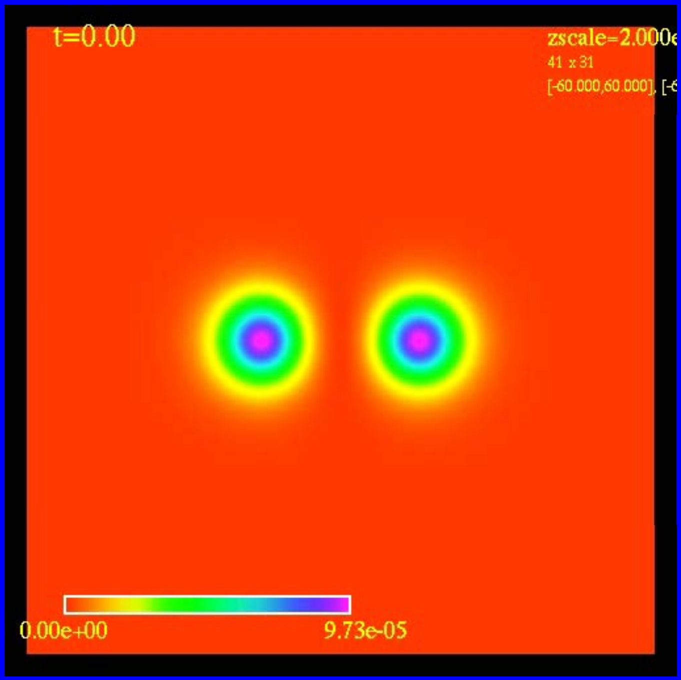

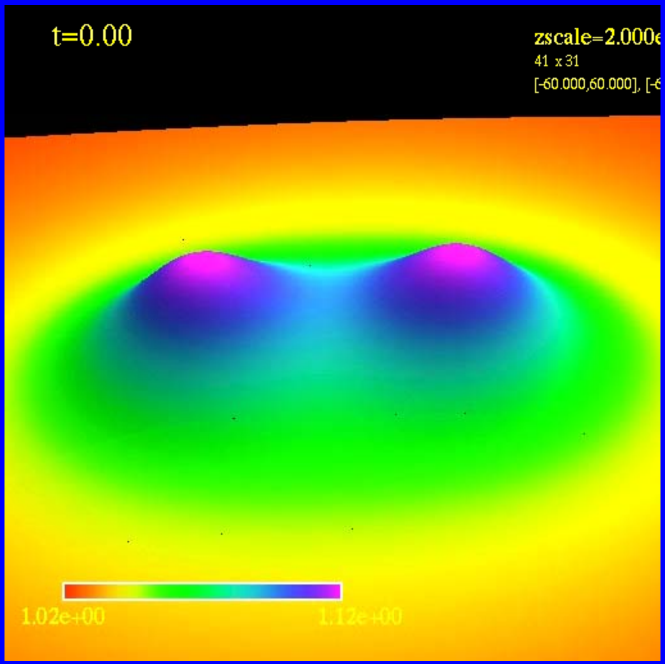

For really orbiting stars, we are constructing initial data and will start evolving them shortly.

Final words

- Present status is really exciting, though we must not hyperventilate…

- Take most things with a grain of salt, while the qualitative picture is more than likely correct, hard quantitative numbers can not be defended yet.

- The new phase in simulations are in the pipeline, AMR-higher order efforts are either just available (e.g. HAD) or so far used infrastructure in the process of being modified.

- If results/behavior are confirmed in general settings, BBH will be:
  - on one hand extremely boring
  - on another extremely tough! Non-linear GR features will have to be extracted from tiny effects in the waves.

Richer systems (BH-NS) or lower frequency (SMBH-compact object) systems might be provide ‘easier’ places to look for non-linear features