GOING BEYOND THE IDEAL APPROXIMATIONS : resistive and anisotropic Ohm law

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Overview

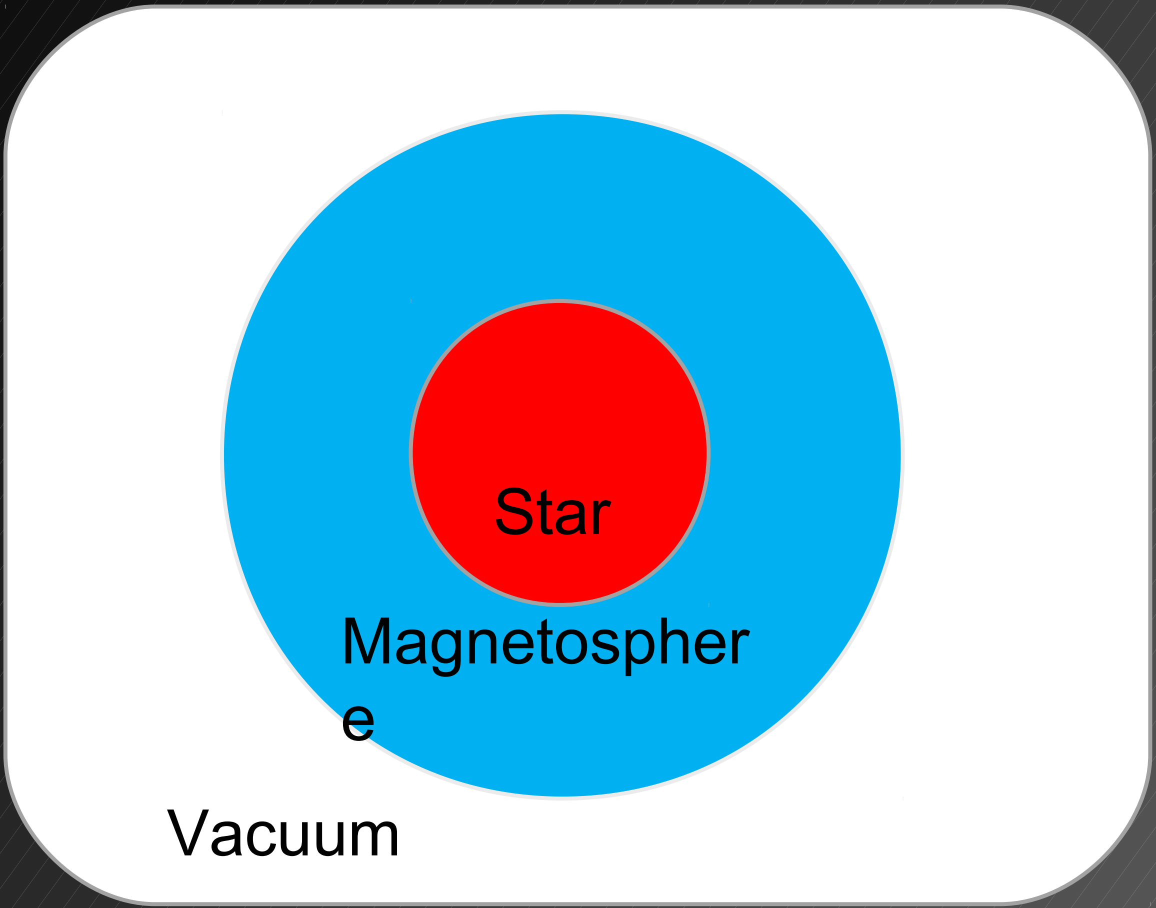
- Motivation
  - Different regimes of the Maxwell equations
  - Resistive effects and anisotropies

- The resistive MHD
  - Approaches to the problem
  - The IMEX Runge-Kutta methods

- Application to the MHD equations

- Summary and conclusions

Different regimes of the Maxwell equations



- Star or disk
  Dominated by the fluid

  IDEAL MHD
- Magnetosphere
  Dominated by the EM

  FORCE FREE
- Vacuum
  no sources

  MAXWELL EQS.

Maxwell equations in 3 regimes

$\partial_t \mathbf{E} - \nabla \times \mathbf{B} = -\mathbf{J}$

$\partial_t \mathbf{B} + \nabla \times \mathbf{E} = 0$

$\nabla \cdot \mathbf{B} = 0$

$\nabla \cdot \mathbf{E} = q$

$\mathbf{J} = \sigma\,(\mathbf{E} + \mathbf{v} \times \mathbf{B})$

σ : conductivity

IDEAL MHD
($\sigma \to \infty$)

$\mathbf{E} = -\mathbf{v} \times \mathbf{B}$

$\partial_t \mathbf{B} - \nabla \times \mathbf{E} = 0$

$\nabla \cdot \mathbf{B} = 0$

FORCE FREE
($qE + \mathbf{J} \times \mathbf{B} = 0$)

$E \cdot B = 0$

$\partial_t B - \nabla \times E = 0$

$\nabla \cdot \mathbf{B} = 0$

MAXWELL EQS.
($\sigma \to 0$)

$\partial_t \mathbf{E} - \nabla \times \mathbf{B} = 0$

$\partial_t \mathbf{B} + \nabla \times \mathbf{E} = 0$

$\nabla \cdot \mathbf{B} = 0$

$\nabla \cdot \mathbf{E} = 0$

Motivation

• The ideal MHD approximation seems to describe properly many astrophysical scenarium (long list), but

  - they may lead to very distorted field lines → reconnections
  - anisotropic effects coming from the Hall term

• The force free approximation describe well the magnetospheres of NS and BHs, but

  - they may lead to current sheets → anomalous resistivity

• Is it possible to have different limits/approximations in the same physical problem?

The relativistic MHD equations (I)

- the description of a fluid in presence of EM fields is given by:

1) Conservation of mass and total energy and momentum + EOS closure relation

Hydrodynamic equations to describe the fluid

ρ: density, u_a: 4-velocity, ε: internal energy, P: pressure

$$\nabla_a (\rho u^a) = 0 \quad , \quad \nabla_a T^{ab} = 0 \quad , \quad P = P(\rho,\varepsilon)$$

$$T_{ab} = [\rho(1+\varepsilon) + P]u_a u_b + P g_{ab} + [F_{ac} F^c{}_b - (F_{cd} F^{cd}) g_{ab}/4]$$

The relativistic MHD equations (II)

2) (Extended) Maxwell equations for the EM fields

$\nabla_a (F^{ab} + g^{ab}\ \Psi) = -I^b + \kappa\ n^b\ \Psi$ F^{ab} : Maxwell tensor

$\nabla_a (*F^{ab} + g^{ab}\ \Phi) = \kappa\ n^a\ \Phi$ I^b : current 4-vector

$\nabla_a I^a = 0$ $I^a = n^a q + J^a$ q : charge, J^a: 3-current

3) The coupling between the fluid and the EM fields, which is given by the choice of current J^i .

The relativistic MHD equations (III)

- 3+1 decomposition (special relativistic)

$$\begin{aligned}
\partial_t \psi + \nabla \cdot \boldsymbol{E} &= q - \kappa\,\psi\,, \\
\partial_t \phi + \nabla \cdot \boldsymbol{B} &= -\kappa\,\phi\,, \\
\partial_t \boldsymbol{E} - \nabla \times \boldsymbol{B} + \nabla \psi &= -\boldsymbol{J}\,, \\
\partial_t \boldsymbol{B} + \nabla \times \boldsymbol{E} + \nabla \phi &= 0\,.
\end{aligned}$$

$$\partial_t \tau + \nabla \cdot \boldsymbol{F}_\tau = 0\,,$$

$$\partial_t \boldsymbol{S} + \nabla \cdot \mathbf{F}_{\boldsymbol{S}} = 0\,,$$

$$\partial_t q + \nabla \cdot \boldsymbol{J} = 0\,,$$

$$\partial_t D + \nabla \cdot \boldsymbol{F}_D = 0$$

$$\begin{aligned}
\tau &\equiv \frac{1}{2}(E^2 + B^2) + h\,W^2 - p \\
\boldsymbol{S} &\equiv \boldsymbol{E} \times \boldsymbol{B} + h\,W^2\,\boldsymbol{v}\,.
\end{aligned}$$

$$\boldsymbol{F}_\tau \equiv \boldsymbol{E} \times \boldsymbol{B} + h\,W^2\,\boldsymbol{v}\,,$$

$$\mathbf{F}_{\boldsymbol{S}} \equiv -\boldsymbol{E}\boldsymbol{E} - \boldsymbol{B}\boldsymbol{B} + hW^2\boldsymbol{v}\boldsymbol{v} + \left[\frac{1}{2}(E^2 + B^2) + p\right]\mathbf{g}\,.$$

$h = \rho(1+\varepsilon) + p$

$W = (1-v^2)^{-1/2}$

The generalized Ohm's law (I)

- The first charge moment of the Boltzmann equation for a two-component fluid (electrons and ions) in the newtonian case

$$\vec{E} = -(\vec{v}\times\vec{B}) + \frac{\vec{j}}{\sigma} + \frac{1}{en_e}\vec{j}\times\vec{B} - \frac{1}{en_e}\nabla.\mathbf{P_e}$$
$$+\frac{m_e}{e^2 n_e}\left\{\frac{\partial\vec{j}}{\partial t} + \nabla.(\vec{\mathbf{v}}\vec{\mathbf{j}} + \vec{\mathbf{j}}\vec{\mathbf{v}} - \frac{1}{en_e}\vec{\mathbf{j}}\vec{\mathbf{j}})\right\}$$

induction, ideal MHD

Ohmic term, allows for dissipation

electron inertia, neglegible

Hall term,introduces anisotropies wrt B

battery

The generalized Ohm's law (II)

- Keep not only the induction term, but also the Ohmic and the Hall ones. In the collision-time approximation, in full GR covariant form

$$I_a = q\, u_a + \sigma^{ab}\, e_a \qquad \sigma^{ab} = \sigma(g^{ab} + \xi^2 b^a b^b + \xi \varepsilon^{abcd}\, u_c\, b_d)$$

$$\xi = e\tau / m \quad , \quad \sigma = n_e e \xi / (1 + \xi^2\, b^2)$$

written in terms of the charge density and EM fields measured by a observer co-moving with the fluid

$$q = -I_a u^a \;,\; e_a \equiv F_{ab}\, u^b \,,\; b_a \equiv F^*_{ab}\, u^b$$

The generalized Ohm's law (III)

- Neglecting the second and third term, in 3+1 form

$$\partial_t \mathbf{E} - \nabla \times \mathbf{B} = -\mathbf{J} = -q\,\mathbf{v} - \sigma W\,[\mathbf{E} + \mathbf{v} \times \mathbf{B} - (\mathbf{E}\cdot\mathbf{v})\,\mathbf{v}]$$

$$\partial_t \mathbf{B} + \nabla \times \mathbf{E} = 0$$

- This is an hyperbolic-relaxation equation, difficult to solve with standard explicit numerical methods

$$\partial_t \mathbf{U} = F(\mathbf{U}) + R(\mathbf{U}) / \varepsilon \qquad \varepsilon = 1/\sigma \text{ relaxation time}$$

- The ideal MHD recovered when $\sigma \rightarrow \infty$, so $E = -v \times B$

$$\partial_t \mathbf{B} - \nabla \times (v \times B) = 0$$

The force free approximation

- From the total energy-momentum conservation and Maxwell equations

$$\nabla_a T^{ab} = 0 \quad \rightarrow \quad \nabla_a T^{ab}{}_{(fluid)} = -\nabla_a T^{ab}{}_{(em)} = -F^{ab} J_a$$

- if $\rho, P \ll B^2$ then $\nabla_a T^{ab}{}_{(fluid)} \ll F^{ab} J_a \approx 0$

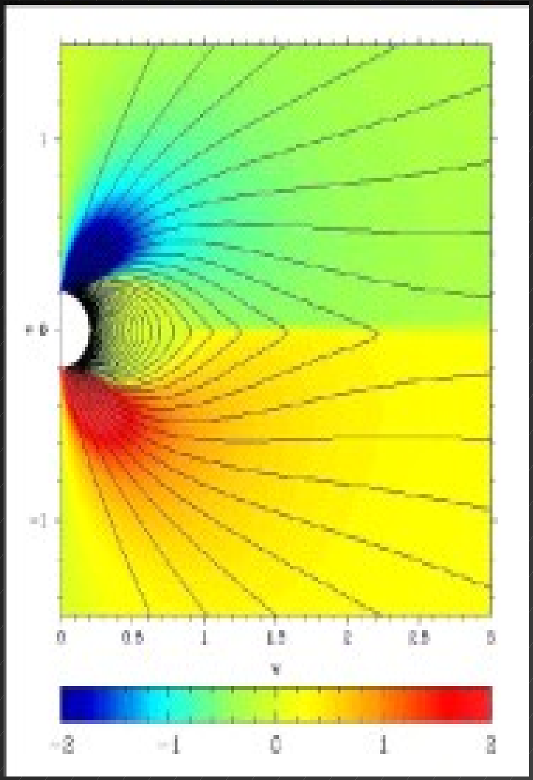
$$\mathbf{E}\cdot\mathbf{J} = 0 \quad , \quad q\,\mathbf{E} + \mathbf{J} \times \mathbf{B} = 0$$

x $\mathbf{B} \rightarrow \quad \mathbf{J} = q\, \mathbf{E}\mathrm{x}\mathbf{B}/\mathbf{B}^2 + (\mathbf{J}\cdot\mathbf{B})\, \mathbf{B}/B^2$

· $\mathbf{B} \rightarrow \quad \mathbf{E}\cdot\mathbf{B} = 0$

$\partial_t(\mathbf{E}\cdot\mathbf{B}) = \mathbf{B}\cdot\nabla \mathrm{x}\mathbf{B} - \mathbf{E}\cdot\nabla \mathrm{x}\mathbf{E} - \mathbf{B}\cdot\mathbf{J} \qquad \partial_t(\mathbf{E}\cdot\mathbf{B}) = 0 \rightarrow \mathbf{B}\cdot\mathbf{J}$

Magnetospheres of NS and BHs with force-free (Komissarov, McKinney & Spitkovski)



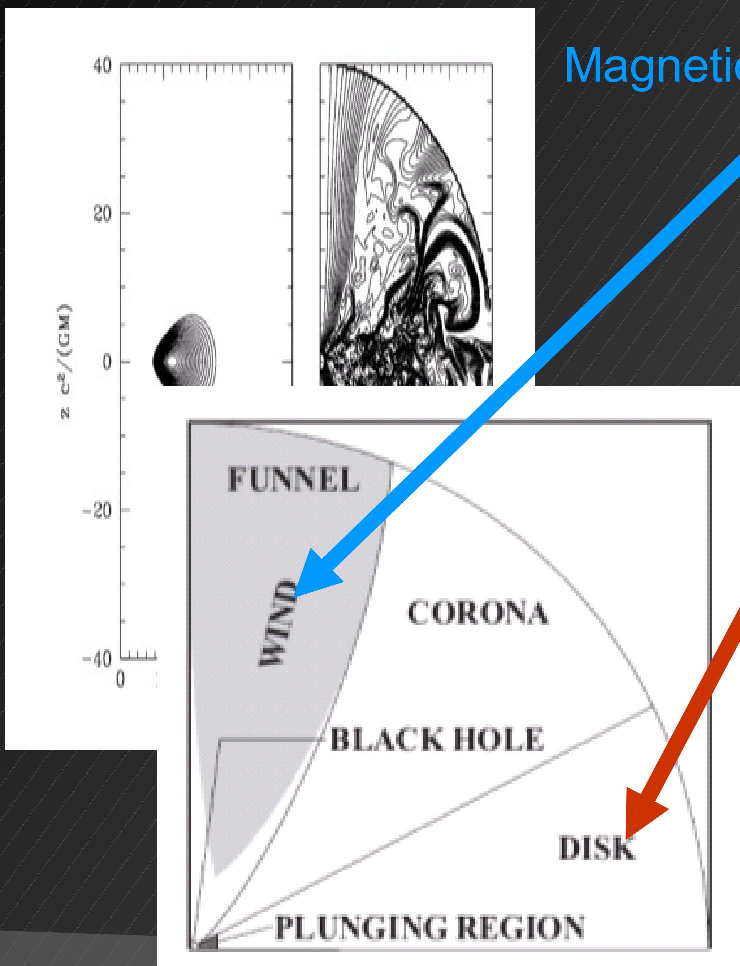
- Current sheet at the equator and instabilities when $B^2-E^2<0$
  $\rightarrow$ inertia effects are not neglegible
  $\rightarrow$ dissipation processes restore E=B

- Let us consider $\mathbf{B}\cdot\mathbf{J}=\sigma\,(\mathbf{E}\cdot\mathbf{B})$

$$\mathbf{J} = [\; q\,\mathbf{E}\mathrm{x}\mathbf{B} + \sigma\,(\mathbf{E}\cdot\mathbf{B})\,\mathbf{B}\;] \,/\, \mathbf{B}^2$$
$$\partial_t(\mathbf{E}\cdot\mathbf{B}) = \ldots - \sigma(\mathbf{E},\mathbf{B})\,(\mathbf{E}\cdot\mathbf{B})$$

- implies $E\cdot B=0$ when $\sigma\rightarrow\infty$

Force-free with ideal MHD BH+disk (McKinney & Gammie)



Magnetically dominated (effectively force-free)
$B^2 >> P$

Matter dominated
$P >> B^2$

the dependence on the Ohm law seems to diminish as $\rho, P \rightarrow 0$, since $F^{ab} J_b = 0$

Resistive MHD

- A complete description of the different regions may be necessary to study magnetized fluid, but it is difficult to match solutions of different limits of the MHD equations

- The equations may lead to very distorted fields, where the limits are not valid anymore and there are significant dissipative effects inside the star or in the current sheets

- Naïve approach : evolve the full Maxwell equations with a generic current prescription in the three domains with no approximations, just changing the effective conductivity. The simplest example is to go from ideal MHD ($\sigma \rightarrow \infty$) to vacuum ($\sigma = 0$).

Resistive MHD

$$\partial_t \mathbf{E} - \nabla \times \mathbf{B} = -q\,\mathbf{v} - \sigma W\,[\,E + \mathbf{v} \times \mathbf{B} - (\mathbf{E}\cdot\mathbf{v})\,\mathbf{v}]$$

$$\partial_t \mathbf{B} + \nabla \times \mathbf{E} = 0$$

$$\partial_t \mathbf{U} = F(\mathbf{U}) + R(\mathbf{U}) / \varepsilon$$

ε (= $1/\sigma$) : relaxation time

Hyperbolic-relaxation equation (STIFF)

difficult to evolve with standard numerical methods

Dealing with the stiff equation

- Approaches to the problem
- The IMEX Runge-Kutta methods

Approaches to the problem

- SOLUTION 1 : let us consider a simple case discretized with an explicit scheme

$$\partial_t \mathbf{U} = F(\mathbf{U}) + R(\mathbf{U}) / \varepsilon \iff \partial_t u = a\, \partial_x u - u / \varepsilon$$

$$(a=0):\ u^{n+1} - u^n = -\Delta t\, u^n / \varepsilon \rightarrow u^{n+1} = u^n (1 - \Delta t/\varepsilon)$$

$$\text{amplification factor } C^n = |u^{n+1} / u^n| < 1 \text{ for stability}$$

- CFL stability condition: $\Delta t < \Delta x / a$
- Stiff stability condition with explicit method: $\Delta t < 2\varepsilon$

if $\Delta t \sim \varepsilon = 1/\sigma \sim 10^{-6}$ → computationally impossible

Approaches to the problem

- SOLUTION 2 : solving the full equation implicitly

- Let us consider an implicit method

$$(a=0): \quad u^{n+1} - u^n = -\Delta t\, u^{n+1} / \varepsilon \;\rightarrow\; u^{n+1} = u^n / (1 + \Delta t/\varepsilon)$$

- Stiff stability condition with implicit method: $\Delta t > 0$
- But… it is expensive/complicated with non-vanishing F(U)

Approaches to the problem

- SOLUTION 3 : the equilibrium system
  - expand the solution around $\varepsilon \to 0$

$$\partial_t \mathbf{U} = F(\mathbf{U}) + R(\mathbf{U}) / \varepsilon \quad \Leftrightarrow \quad \partial_{tt} B - \Delta \mathbf{B} = [-\partial_t \mathbf{B} + \nabla \times (\mathbf{v} \times \mathbf{B})] / \varepsilon$$

$$\mathbf{U} = \mathbf{U}_0 + \varepsilon \mathbf{U}_1 + O(\varepsilon^2) \qquad \mathbf{B} = \mathbf{B}_0 + \varepsilon \mathbf{B}_1 + O(\varepsilon^2)$$

$$\Downarrow$$

$O(\varepsilon^0)$: IDEAL MHD $\quad \partial_t \mathbf{B_0} - \nabla \times (\mathbf{v} \times \mathbf{B_0}) = 0$

$O(\varepsilon^1)$: $\quad \partial_t \mathbf{B_1} - \nabla \times (\mathbf{v} \times \mathbf{B_1}) = -(\partial_{tt} \mathbf{B}_0 - \Delta \mathbf{B}_0)$

- hierarchy of solutions : compute $\mathbf{B}_0$, then $\mathbf{B}_1$,… but it is only valid close to $\varepsilon \to 0$

Approaches to the problem

- SOLUTION 4 : Strang Splitting

$$\partial_t \mathbf{U} = F(\mathbf{U}) + R(\mathbf{U}) / \varepsilon \iff \partial_t \mathbf{U} = S(\Delta t/2) ¤ T(\Delta t) ¤ S(\Delta t/2)\, \mathbf{U}$$

$$U^* \quad : \quad U^* = U^n + (\Delta t/2)\, R(U^n) / \varepsilon$$

$$U^{**} \quad : \quad U^{**} = U^* + \Delta t\, F(U^*)$$

$$U^{n+1} \quad : \quad U^{n+1} = U^{**} + (\Delta t/2)\, R(U^{**}) / \varepsilon$$

- The source step can be solved exactly with the analytical solution (Komissarov 2007)… but it does not work for strong stiff terms in the presence of shocks

Approaches to the problem

- SOLUTION 5 : discontinuous Galerkin methods

- There are high order schemes (3-5th order) which can deal with the stiff terms (Dumbser & Zanotti 2009)… but it is complicated and expensive

The IMEX Runge Kutta methods

- treat implicitly the stiff part and explicitly the non-stiff (IMplicit-EXplicit methods)

$$\partial_t \mathbf{U} = F(\mathbf{U}) + R(\mathbf{U}) / \varepsilon$$

$$U^{(i)} = U^n + \Delta t \, \Sigma \, \underline{a}_{ij} \, F(U^{(j)}) + \Delta t \, \Sigma \, a_{ij} \, R(U^{(j)}) / \varepsilon$$

$$U^{n+1} = U^n + \Delta t \, \Sigma \, \underline{\omega}_i \, F(U^{(i)}) + \Delta t \, \Sigma \, \omega_i \, R(U^{(i)}) / \varepsilon$$

Explicit RK

$\underline{c}_1$	0	0	0	….
$\underline{c}_2$	$\underline{a}_{12}$	0	0	….
…	…………………			
$\underline{c}_n$	$\underline{a}_{1n}$	$\underline{a}_{2n}$	… 0	
	$\underline{\omega}_1$	$\underline{\omega}_2$	… $\underline{\omega}_n$	

DIRK

c_1	a_{11}	0	0	….
c_2	a_{12}	a_{22}	0	….
…	…………………			
c_n	a_{1n}	a_{2n}	… a_{m}	
	ω_1	ω_2	… ω_n	

Butcher Tableau

The IMEX Runge Kutta methods

- Let us consider a simple IMEX RK as an example

$$\partial_t \mathbf{U} = F(\mathbf{U}) + R(\mathbf{U}) / \varepsilon$$

IMEX-Midpoint(1,2,2)

0	0	0		0	0	0
1/2	1/2	0		1/2	0	1/2
	0	1			0	1

$$\mathbf{U}^1 = \mathbf{U}^n$$

$$\mathbf{U}^2 = \mathbf{U}^n + \Delta t\, F(\mathbf{U}^1)/2 + \Delta t\, R(\mathbf{U}^2)/(2\,\varepsilon)$$

$$U^{n+1} = U^n + \Delta t\, F(U^2) + \Delta t\, R(U^2) / \varepsilon$$

- only the stiff part has to be inverted
- high order convergence in time (usually 3 order)
- strong theoretical background (it has to work!)

Application to the Maxwell equations

- Inverting explicitly the stiff part
- Numerical tests

The relativistic MHD equations

- the conserved variables (D,τ,S^i,E^i,B^i,q) are evolved by using HRSC methods for conservation laws

- the primitive variables ($\rho,\varepsilon,P,v^i,E^i,B^i,q$) are needed to compute the rhs of the evolution equations

-The transformation from conserved to primitive variables is non-linear and has to be solved numerically in general

→ weakest link of any relativistic MHD code!

Inverting explicitely the stiff part

- only the evolution of the electric field has stiff terms

$$\partial_t \mathbf{E} - \nabla \times \mathbf{B} = q\,\mathbf{v} - \sigma W [\, \mathbf{v} \times \mathbf{B} + \mathbf{E} - (\mathbf{E}\cdot\mathbf{v})\, \mathbf{v}]$$

- use standard TVD RK methods for the other fields and apply the IMEX only to E

$$\partial_t \mathbf{U} = F(\mathbf{U}) + R(\mathbf{U}) / \varepsilon \longrightarrow \begin{aligned} F(\mathbf{E}) &= \nabla \times \mathbf{B} + q\,\mathbf{v} \\ R(\mathbf{E}) &= - W [\mathbf{v} \times \mathbf{B} + \mathbf{E} - (\mathbf{E}\cdot\mathbf{v})\, \mathbf{v}] \\ S &= - W\, \mathbf{v} \times \mathbf{B} \end{aligned}$$

Inverting explicitly the stiff part

Example:

$$\mathbf{U}^1 = \mathbf{U}^n$$

$$\mathbf{U}^2 = \mathbf{U}^n + \Delta t\, F(\mathbf{U}^1)/2 + \Delta t\, R(\mathbf{U}^2)/(2\,\varepsilon)$$

$$U^{n+1} = U^n + \Delta t\, F(U^2) + \Delta t\, R(U^2)/\varepsilon$$

- compute the explicit part

$$\mathbf{E}^* = \mathbf{E}^n + \Delta t\, F(\mathbf{E}^1)/2$$

- invert explicitly the implicit part

$$E^2 = M(v, B)\, [\, E^* + \Delta t\, S / (2\,\varepsilon)\,]$$

- compute $F(E^2)$ and $R(E^2)$ to update E^{n+1}

The relativistic MHD equations

- the conserved variables (D,τ,S^i,E^i,B^i,q) are evolved by using HRSC methods for conservation laws

- the primitive variables $(\rho,\varepsilon,P,v^i,E^i,B^i,q)$ are needed to compute the rhs of the evolution equations

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→ weakest link of any relativistic MHD code!

Test 1: the Alfven wave (del Zanna 2007)

- Testing the high co

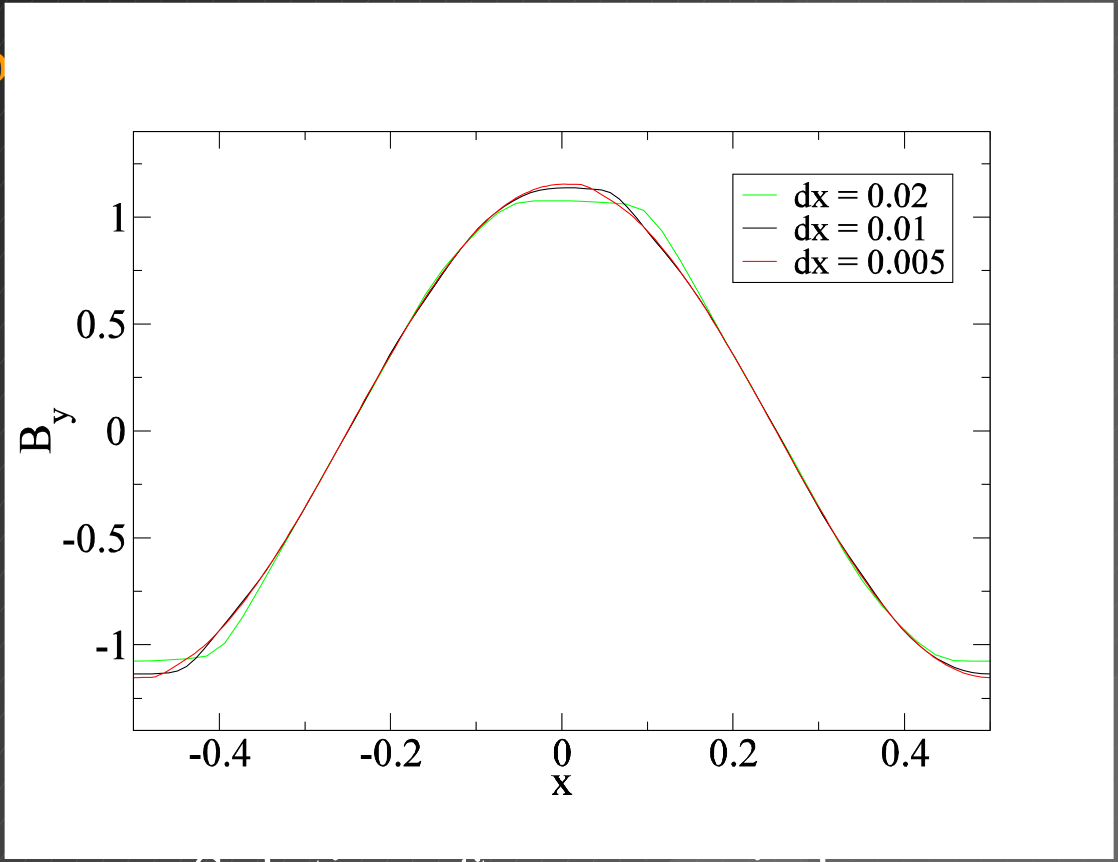
$B_y = B_o \cos(x - v_A t)$

$B_z = B_o \sin(x - v_A t)$

$v_y = -v_A B_y/B_o$

$v_z = -v_A B_z/B_o$

Alfven speed v_A



$P = \rho = 1$, $v_A = 1/2$

conductivity $\sigma = 10^6$

Solution after one period
(periodic boundary conditions)

Test 2: the current sheet (Komissarov 2007)

- Testing the low conducti

$P=cte, \rho=cte$

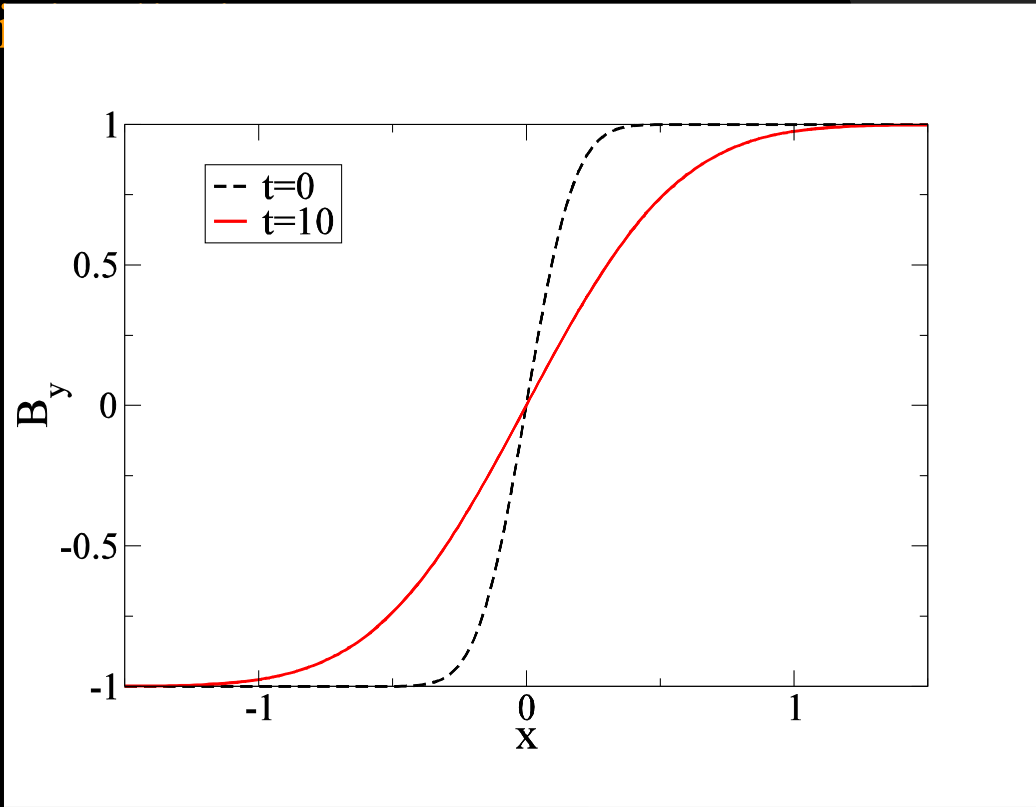
$E = v = 0$

$B= (0,B_y(x,t),0)$

$\partial_t B_y - (1/\sigma)\, \partial_{xx} B_y = 0$

$B_y = B_o\, \mathrm{erf}[(\sigma/(4\,\xi))^{1/2}]$

with $\xi=t/x^2$



Solution at t=10 with σ=100

Test 3: the shock tube problem

- Testing the resistive MHD with shocks

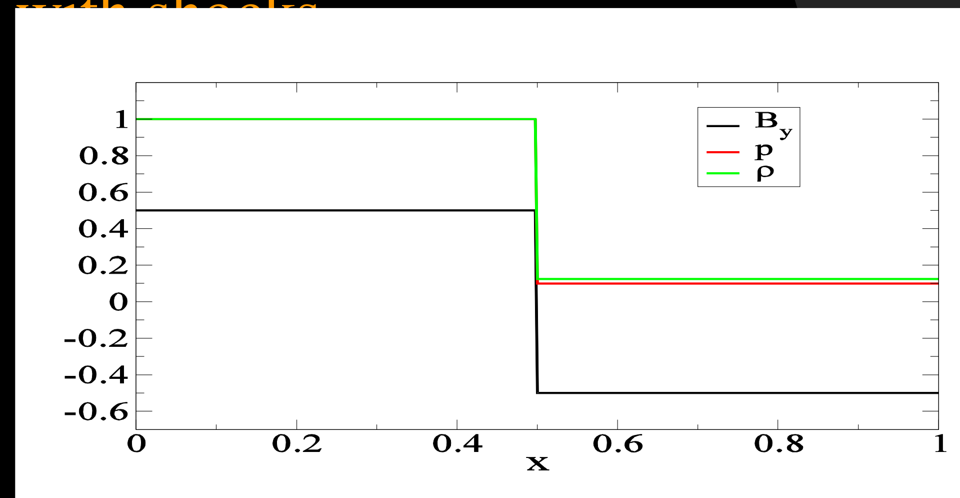
Left state

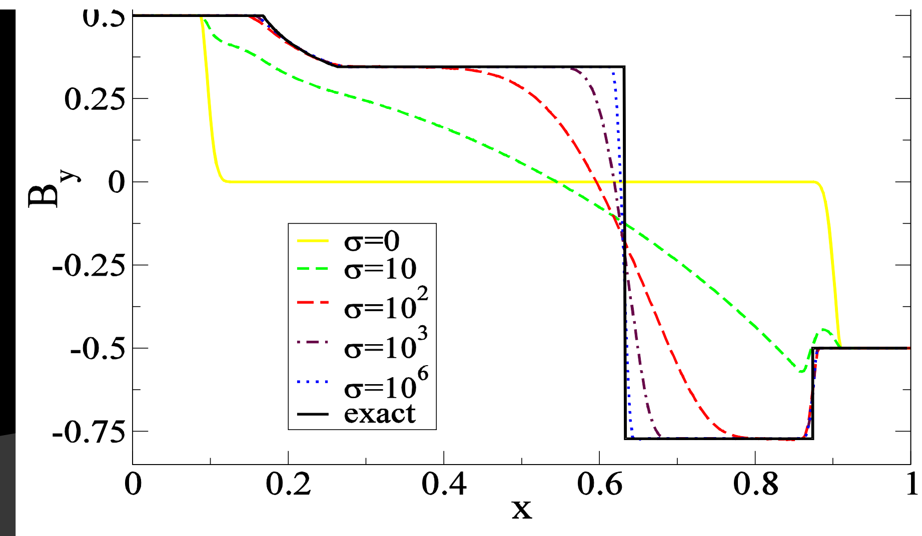
$(\rho^L, p^L, B_y{}^L) = (1, 1, 1/2)$

Right state

$(\rho^R, p^R, B_y{}^R) = (1/8, 0.1, -1/2)$

Solution at t=0.4





Test 4: the cylindrical explosion

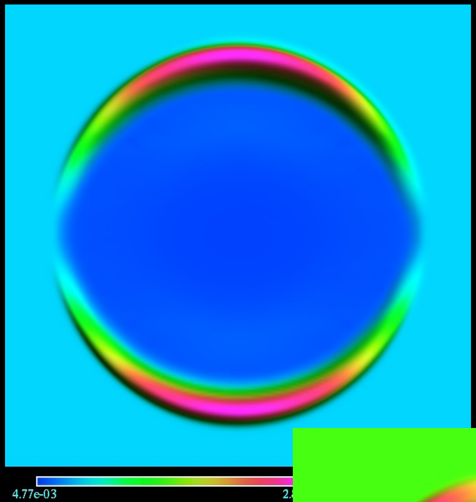
- Testing the resistive MHD with shocks in 2D

$r<0.8 \quad p=1,\ \rho = 0.01$

$r>1.0 \quad p= \rho = 0.001$

$B = (0.05, 0, 0)$

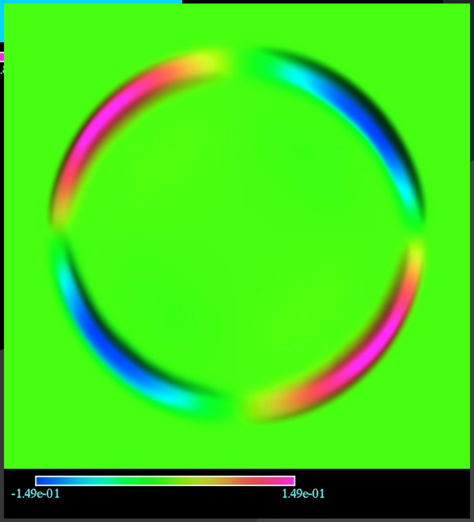
$E = q = 0$



B_x

Solution at t=4

B_y



Test 5: the cylindrical star

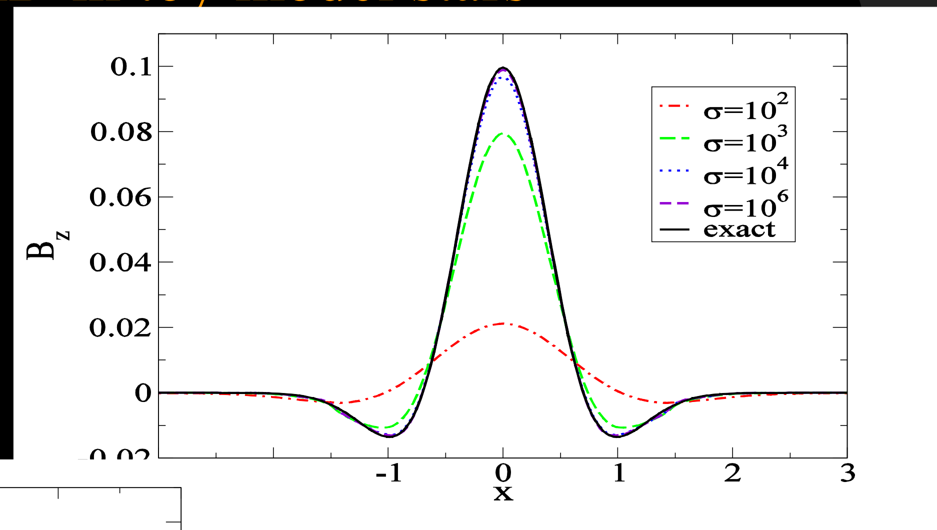
- Testing the resistive MHD in toy model stars

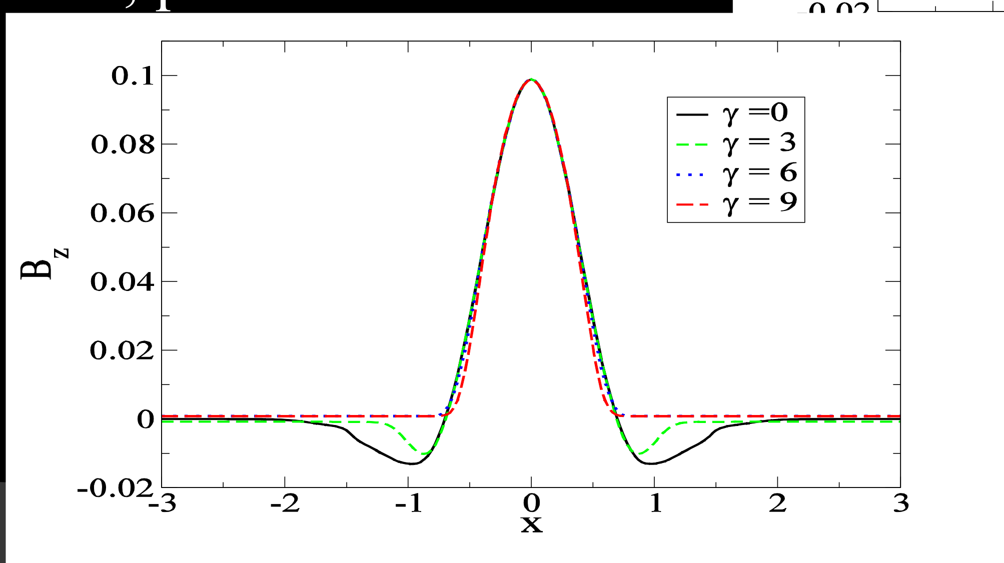
$\rho = \rho_0 \exp[-(r/r_o)^2]$

$v_\varphi = \rho\,\Omega$

$B_z = 2\,B_o[1 - (r/r_o)^2]$

E,q from ideal MHD





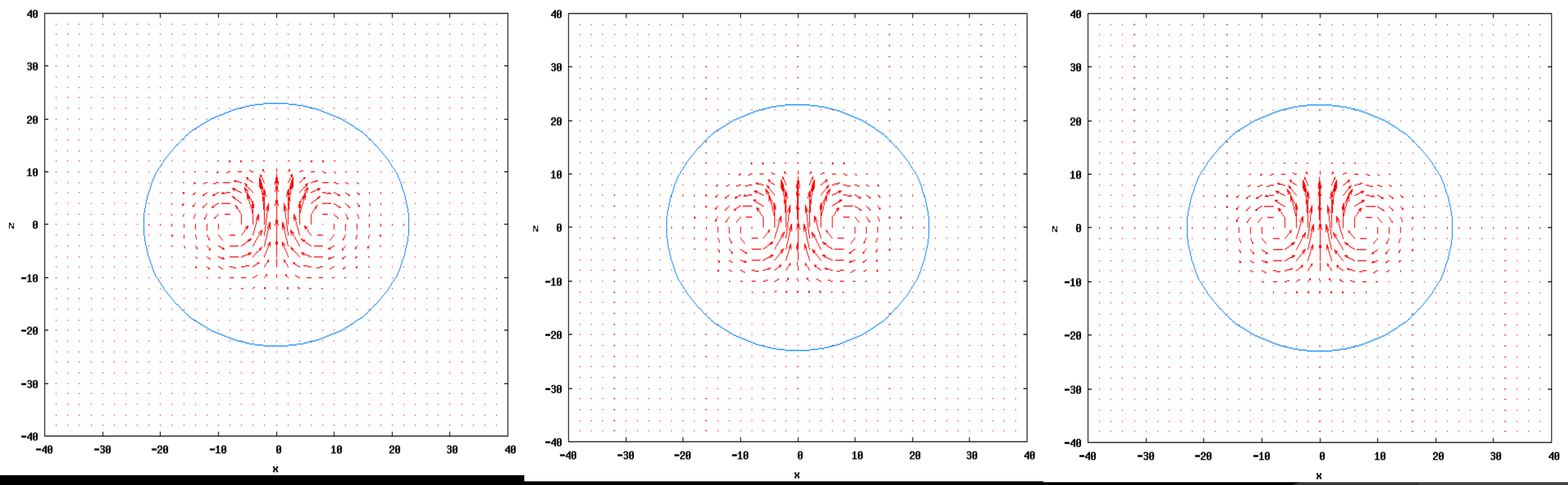
$\sigma = \sigma_0\, D^\gamma \qquad D = \rho W$

Extension to general relativity

$$
\begin{aligned}
\partial_t(\sqrt{\gamma}\, B^i) + \partial_k[-\beta^k \sqrt{\gamma}\, B^i + \alpha\epsilon^{ikj} \sqrt{\gamma}\, E_j\,] &= \\
& -\sqrt{\gamma}\, B^k(\partial_k\beta^i) - \alpha\sqrt{\gamma}\,\gamma^{ij}\partial_j\phi \\
\partial_t(\sqrt{\gamma}\, E^i) + \partial_k[-\beta^k \sqrt{\gamma}\, E^i - \alpha\epsilon^{ikj} \sqrt{\gamma}\, B_j\,] &= \\
& -\sqrt{\gamma}\, E^k(\partial_k\beta^i) - \alpha\sqrt{\gamma}\,\gamma^{ij}\partial_j\Psi - 4\pi\alpha\sqrt{\gamma}\, J^i \\
\partial_t\phi + \partial_k[-\beta^k\phi + \alpha B^k\,] &= \\
& -\phi\,(\partial_k\beta^k) + B^k(\partial_k\alpha) - \alpha\Gamma^i_{ki}B^k - \alpha\kappa\phi \\
\partial_t\Psi + \partial_k[-\beta^k\Psi + \alpha E^k\,] &= \\
& -\Psi\,(\partial_k\beta^k) + E^k(\partial_k\alpha) - \alpha\Gamma^i_{ki}E^k + 4\pi\alpha q - \alpha\kappa\Psi \\
\partial_t(\sqrt{\gamma}\, q) + \partial_k[-\beta^k\sqrt{\gamma}\, q + \alpha\sqrt{\gamma}\, J^k\,] &= 0 \\
\partial_t(\sqrt{\gamma}\, D) + \partial_k[\,\sqrt{\gamma}\, D\,(\alpha v^k - \beta^k)\,] &= 0 \\
\partial_t(\sqrt{\gamma}\,\tau) + \partial_k[\,\sqrt{\gamma}\,(\alpha S^k - \beta^k\tau)\,] &= \sqrt{\gamma}\,[\,\alpha S^{ij}K_{ij} - S^j\partial_j\alpha\,] \\
\partial_t(\sqrt{\gamma}\, S_i) + \partial_k[\,\sqrt{\gamma}\,(\alpha S^k{}_i - \beta^k S_i)\,] &= \sqrt{\gamma}\,[\,\alpha\Gamma^j_{ik}S^k{}_j + S_j\,\partial_i\beta^j - \tau\partial_i\alpha\,]
\end{aligned}
$$

Extension to general relativity

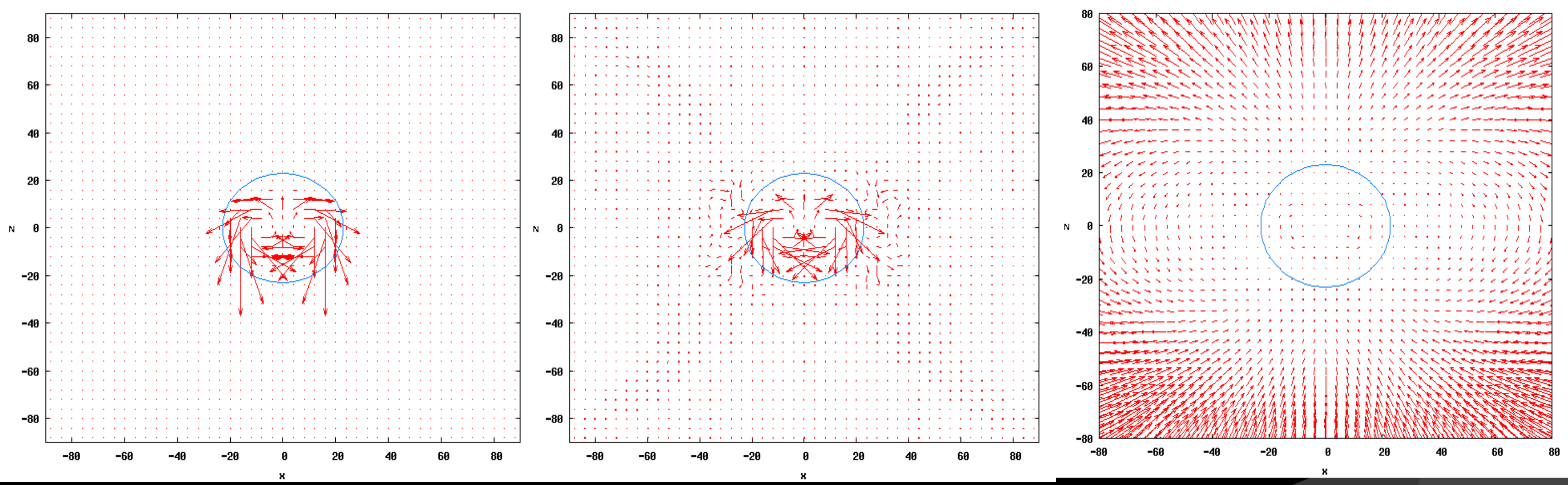
- Rotating neutron star with a poloidal magnetic field



t =0 after 2 periods $\sigma = \sigma_0 = 10^6$ $\sigma = \sigma_0 \rho^2$

Extension to general relativity

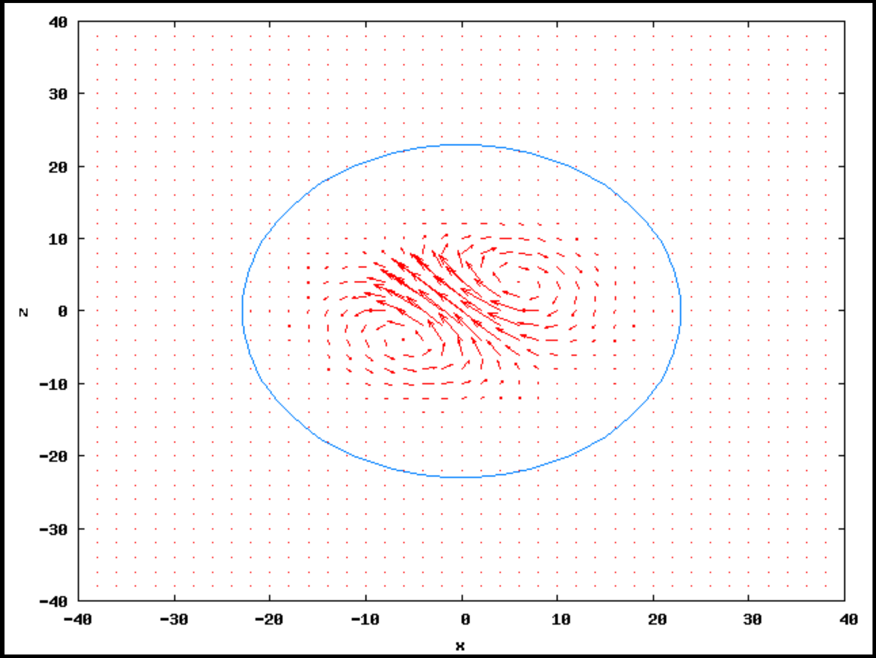
- plot r^2B to show the outer region

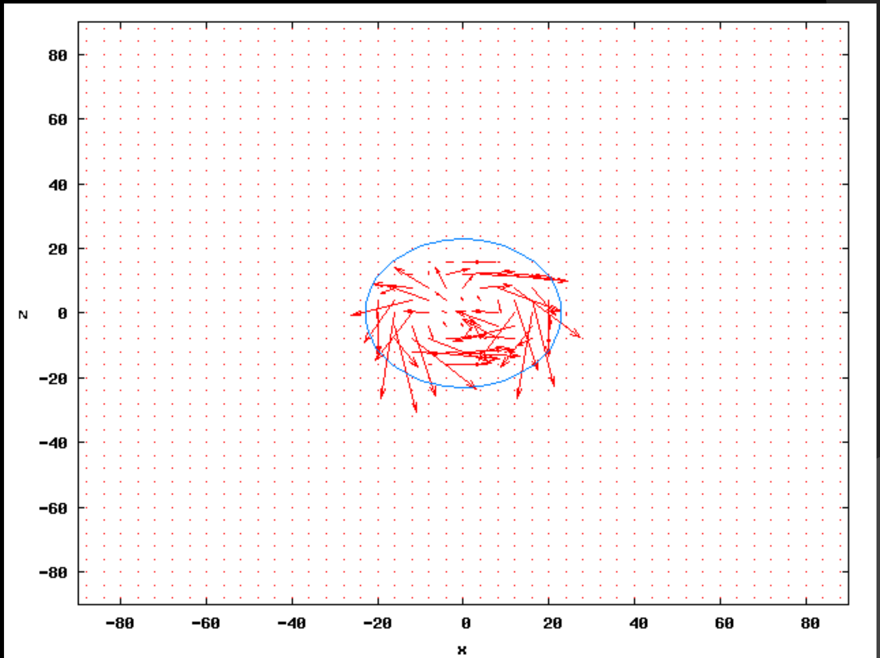


t =0 after 2 periods $\sigma = \sigma_0 = 10^6$ $\sigma = \sigma_0\,\rho^2$

Extension to general relativity

- Rotating neutron star with a poloidal field disaligned 45^o





t=0 B r^2 B

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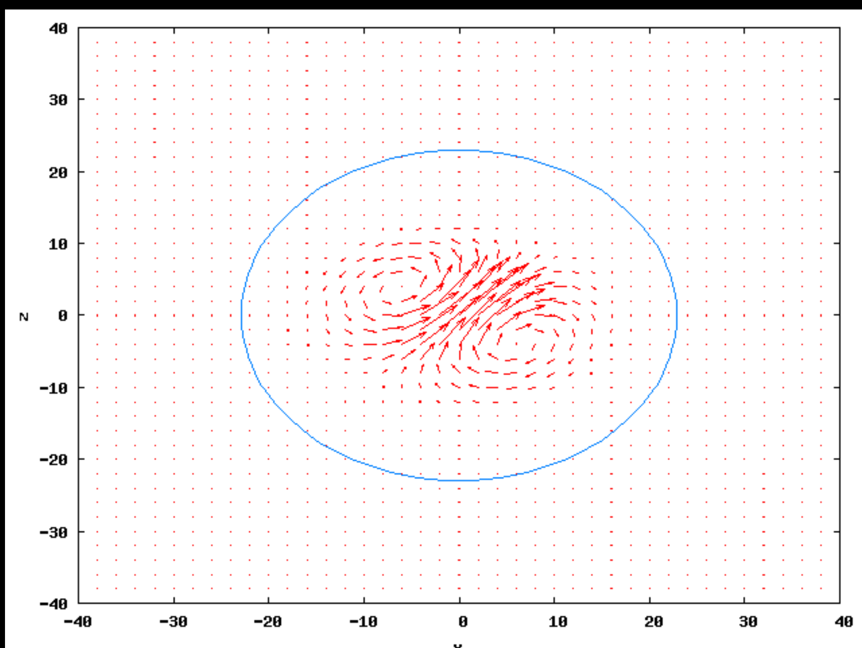

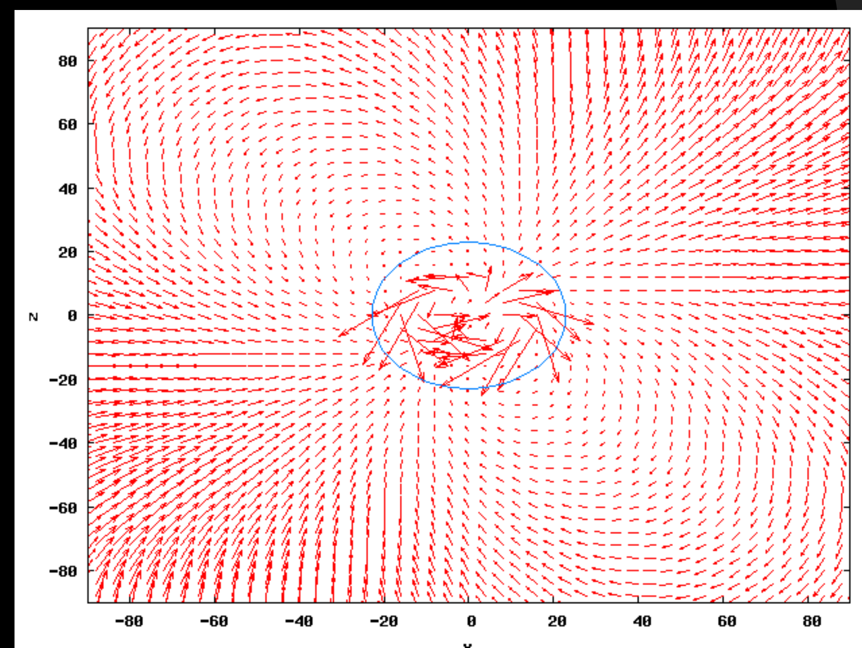

t=1.5P B r^2 B

Summary and conclusions

- the IMEX Runge-Kutta allows to solve the resistive-anisotropic MHD equations in different regimes

- the IMEX