Num Rel beyond vacuum scenarios

(with an eye towards astro apps)

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Requirements/issues

- Fluids:
  - develop shocks/discontinuities for generic data
  - develop further structure/smaller scales
  - might have their own constraints to satisfy
- Physics
  - Incorporate relevant physics processes.
  - Tailor techniques to be able to extract physics
- Numerical techniques?
  - Finite Differences; Finite Elements; Spectral methods?

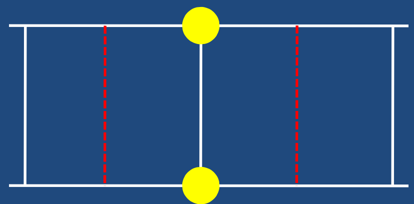
timeline

- NS-NS
  - *Shibata,Uryu, etc > 2002 (BSSN,equat symmetry,uniform grid), M?*
    - *Equal mass, non-equal mass, polytrope with different Γ, Latymer-Swesty eqn of state.*
  - *Miller-Suen… 2004-2007 (BSSN, equat symmetry, FMR) no M*
    - *Equal mass. Analyzed influence of close boundaries*
  - *Duez-Marronetti-Shapiro-Baumgarte. no M*
    - *Equal mass , no conservation form of hydro eqns*
    - *New efforts. Axysimmetry. with M*
  - *Anderson-Palenzuela-Motl-LL-Hirschmann-Neilsen-Liebling-Tohline (GRharmonic,3D,AMR)*
    - *Equal mass, eccentricity, M*

- BH-NS
  - *Shibata-Uryu, 2007. (BSSN,equat symmetry,unif grid) no M*
  - *Baiotti-Rezolla. (BSSN, FMR, M?)*
  - Duez-Kidder-Teukolsky (GHarmonic,full 3D, PSpectral+Fdiff)
  - Anderson-LL-Neilsen-Palenzuela-LL (GHarmonic,3D,AMR,M)

Numerical techniques

- Fluid eqns.
  - Use high resolution shock capturing schemes. CENO + PPM



$$u_{,t} + \partial_i f^i = s(u)$$

$$u^i{}_{,t} = \frac{\hat{f}^{i+1/2} - \hat{f}^{i-1/2}}{dx} + s(u^i)$$

$$f^{HLL} = \frac{\lambda_r^+ f(u^l) - \lambda_r^- f(u^r) + \lambda_r^+ \lambda_r^- (u^r - u^l)}{(\lambda_r^+ - \lambda_l^-)}$$

MHD eqns. Simple example

- $\alpha = 0$: conservative, but weakly hyperbolic
- $\alpha = 1$: non-conservative, but hyperbolic!
- c_l ; s drivers of constraint

$$\partial_t \rho = -\nabla_i(\rho v^i)$$
$$\rho \partial_t v^i = -\rho v^j \nabla_j v^i - \nabla^i p - B_k(\nabla^i B^k - \nabla^k B^i) - \alpha B^i \nabla_k B^k$$
$$\partial_t B^i = -\nabla_j(u^j B^i - u^i B^j) - \alpha u^i \nabla_j B^j - c_l \nabla^i \phi$$
$$\partial_t e = -\nabla_i((e + p + \frac{1}{2}B^2)v^i - B^i v \cdot B) - \alpha v^i B_i \nabla_k B^k - c_l B^k \nabla_k \phi$$
$$\partial_t \phi = -\alpha u^j \nabla_j \phi - c_l \nabla_j B^j - s\phi$$

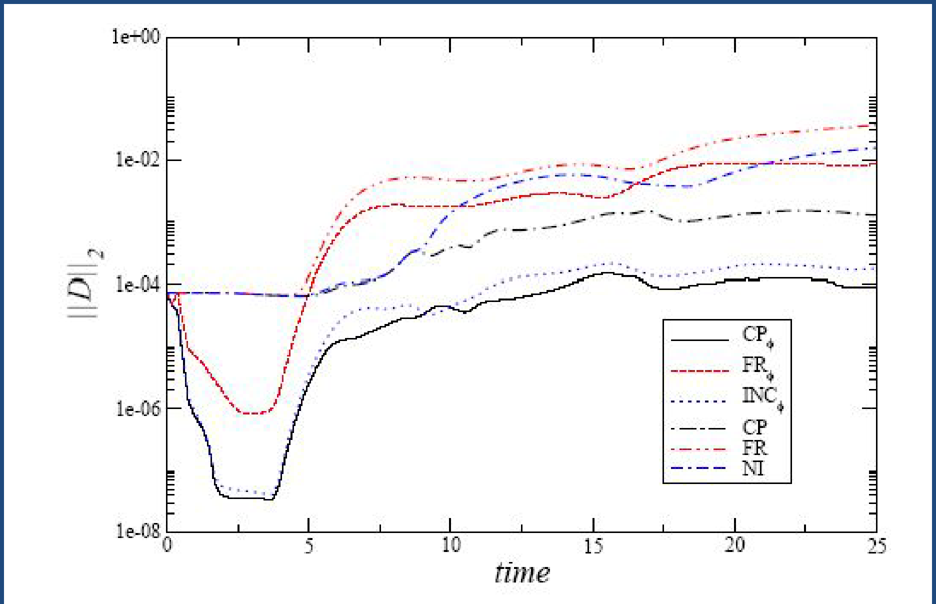
where

$$p := (\gamma - 1)(e - \frac{1}{2}\rho v^2 - \frac{1}{2}B^2) \qquad c_s^2 := \gamma p / \rho$$

Constraint control

Boundary conditions

Flexibility of picking algorithms (i.e. not tied to constraint transport)



[Cecere,LL,Reula 07]

Numerical techniques. GR eqns

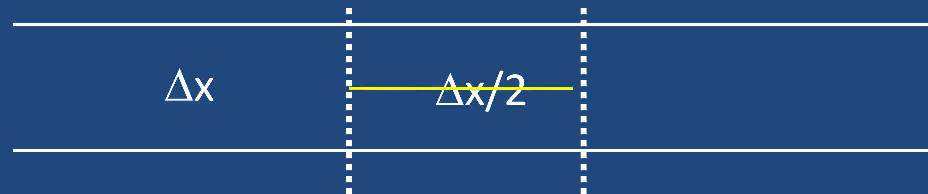
- Guiding principle: reproduce analytical steps at 'all' cost. $u_{,t} = A^i(x,t)u_{,i} + B(x,t)u$
  - What's involved here? $E = \int u F \bar{u}\, dv \Rightarrow \dot{E} \le aE + b$
  - Why did we get this? $\int \dot{u}u \to \int u_{,x}u \to u^2 |_a^b$
- Operator? $\partial \to D \text{ s.t. } \quad (u, Du) = -(Du, u) + u^2$
- Boundaries? $(u, ADu) = \sum_i u_i ADu|_i = F\{A^x/\Delta; (A^x + A^y)/\Delta; (A^x + A^y + A^z)/\Delta\}$
- Dissipation? $RHS \to RHS + L(u) \quad s.t. \ (u, Lu) \le 0$
- Integration in time. RK3-RK4 preserves the discrete energy → stability!

Gustaffson-Kreiss-Oliger; Strand; Olsson; Tadmor → Calabrese-L.L.-Neilsen-Pullin-Reula-Sarbach-Tiglio

Don't want AMR spoiling anything

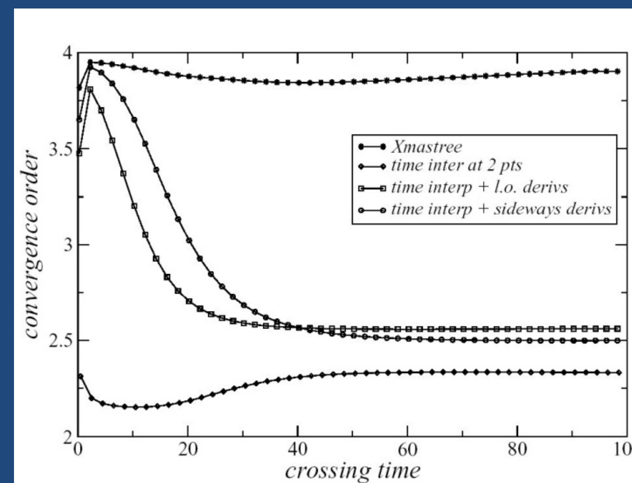
$$u_p^{n+1} = Q\, u_p^n$$

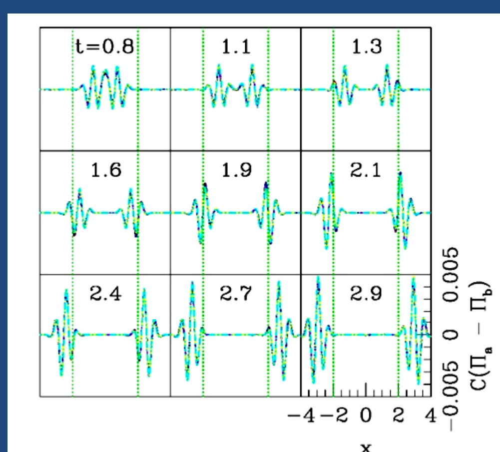
$$Q \equiv (1-P)U + P(J_{21}\, B\,U\, B\,U\, J_{12})$$

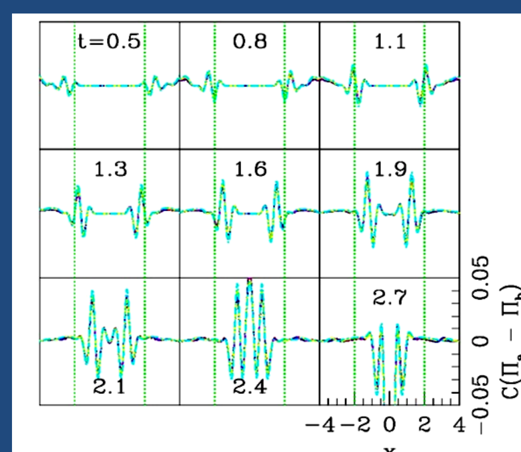


- 2^{nd} order . OK stability & accuracy

- Higher order?
  - Unstable unless dissipation is used
  - Convergence order can be affected







[LL,Liebling,Reula]

HAD

- Distributed AMR via MPI
  - Vertex and cell centered
  - Couple various projects together
  - Black hole excision
  - Surface Extraction
  - *Tapered* AMR boundaries
  - Elliptic solves
  - Public Release: `(http://had.liu.edu)`
- Projects:
  - MHD (vertex-centered)
  - HD (cell-centered)
  - GR…harmonic
  - GR…ADM
  - Various scalar Field models

Extraction of waves

- Two techniques…. Both cavalier so far
  - Perturbative approach (Zerilli,Teukolsky) requires singling out a background.
  - Weyl scalars. Assumes asymptotic structure holds at finite distances, ignores gauge issues.

- Weyl scalars

$$\Psi_4 = C_{abcd}\, n^a \overline{m}^b n^c \overline{m}^d$$

$$\Psi_2 = C_{abcd}\, l^a m^b \overline{m}^c n^d$$

$$\sigma = m^a m^b \nabla_a l_b$$

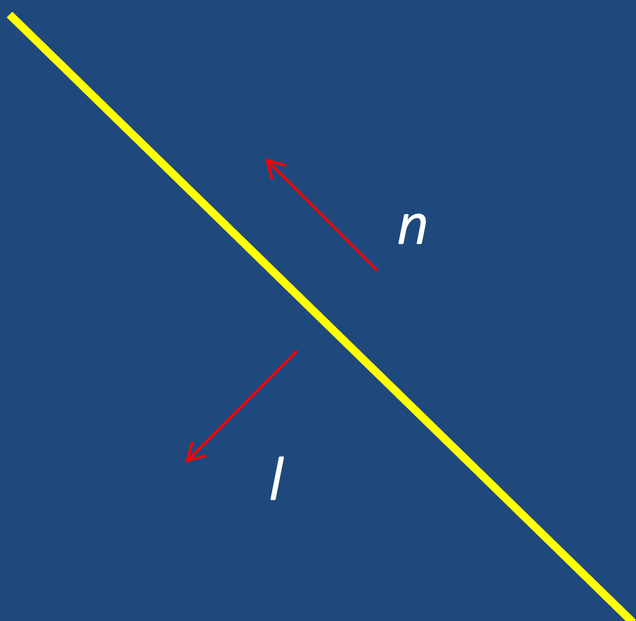
$$\downarrow$$

$$\Psi_4^0 = -\overline{\sigma}_{,uu}$$

$$\downarrow$$

$$M - \frac{1}{4\pi}\int (\Psi_2 + \overline{\sigma}_{,u}\sigma)\, d\Omega$$

$$\dot{M} = -\frac{1}{4\pi}\int \overline{\sigma}_{,u}\sigma_{,u}\, d\Omega$$



- What's the deal in numerical relativity?
  - Tetrad (l,n,m,m^*) is defined at a finite distance. $X^2 = R^2$
    - *Background metric?*
  - Induced angular metric $g = S + C/r$ with S *the unit sphere metric*
    - *Angular part only conformal to unit sphere metric $S = q\, V^{-2}$*
  - $g_{uu} = 1$; $g_{uA} = 0$ (inertial observers stay at const angles; clocks tick the same)

Corrections?

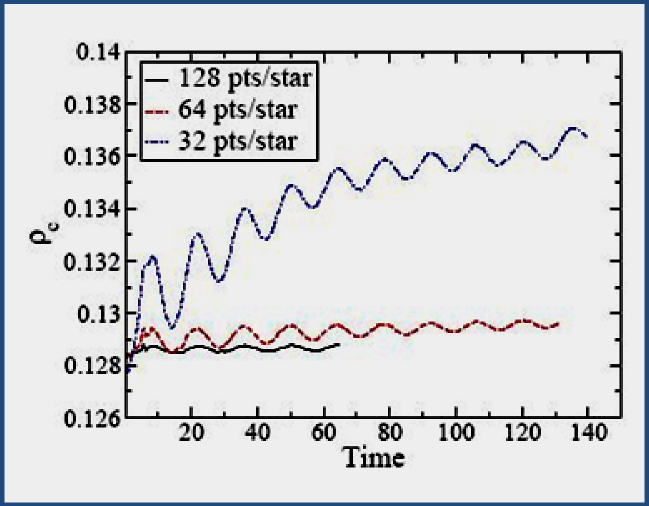
- simplest case ($g_{uA} = 0$)

$$\Psi_4 = \mathit{\Psi}_4 / (V^3\, g_{ur}{}^2)$$

[LL-Moreschi 07]

Putting all pieces together

- Single TOV star. Static solution
    - Truncation errors introduce perturbations
    - compare with known values



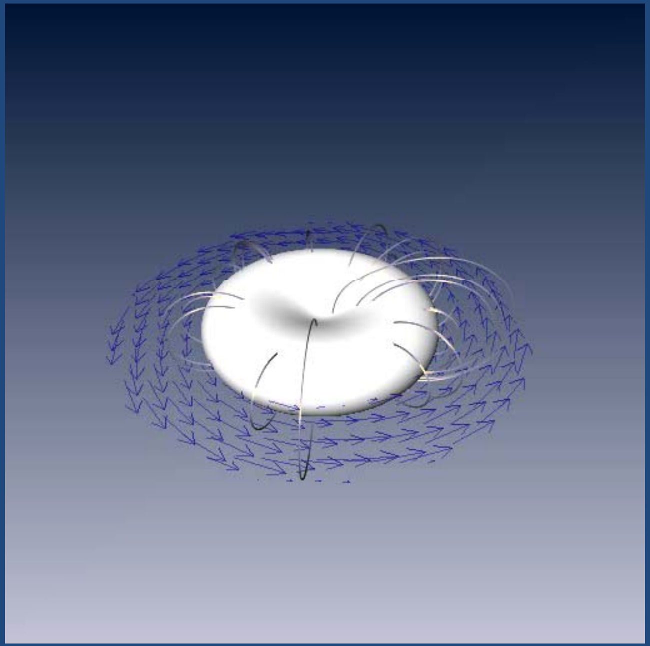
Mode	3D GRHD code (kHz)	Perturbation results (kHz)	Relative Difference (%)
F	14.01 ± 0.61	14.42	2.88
H1	39.59 ± 1.01	39.55	0.1
H2	59.89 ± 1.02	59.16	1.2
H3	76.94 ± 1.83	77.76	1.1

[Anderson,Hirschmann,LL,Liebling,Neilsen,Palenzuela]

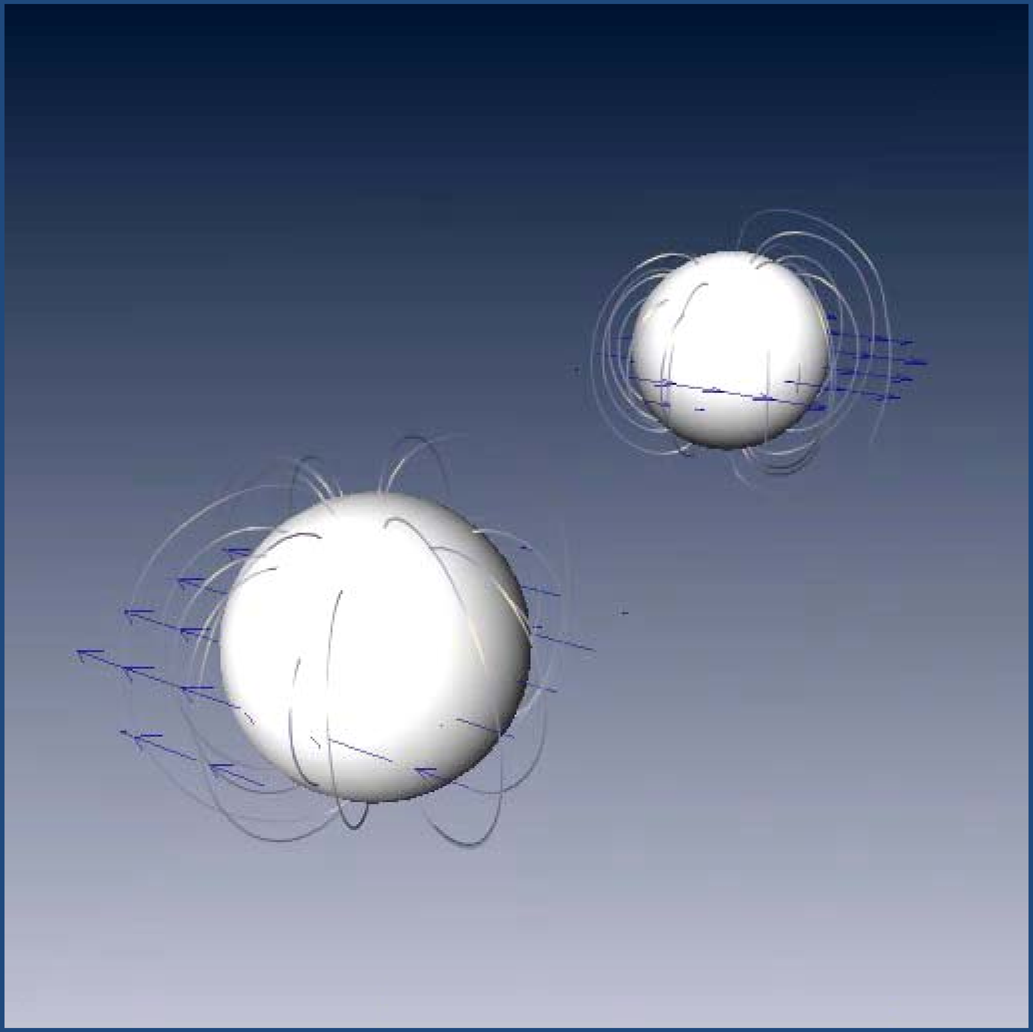
Adding magnetic fields

- Not differentially rotating, magnetic fields don’t play a significant role
  - Instability doesn’t take off

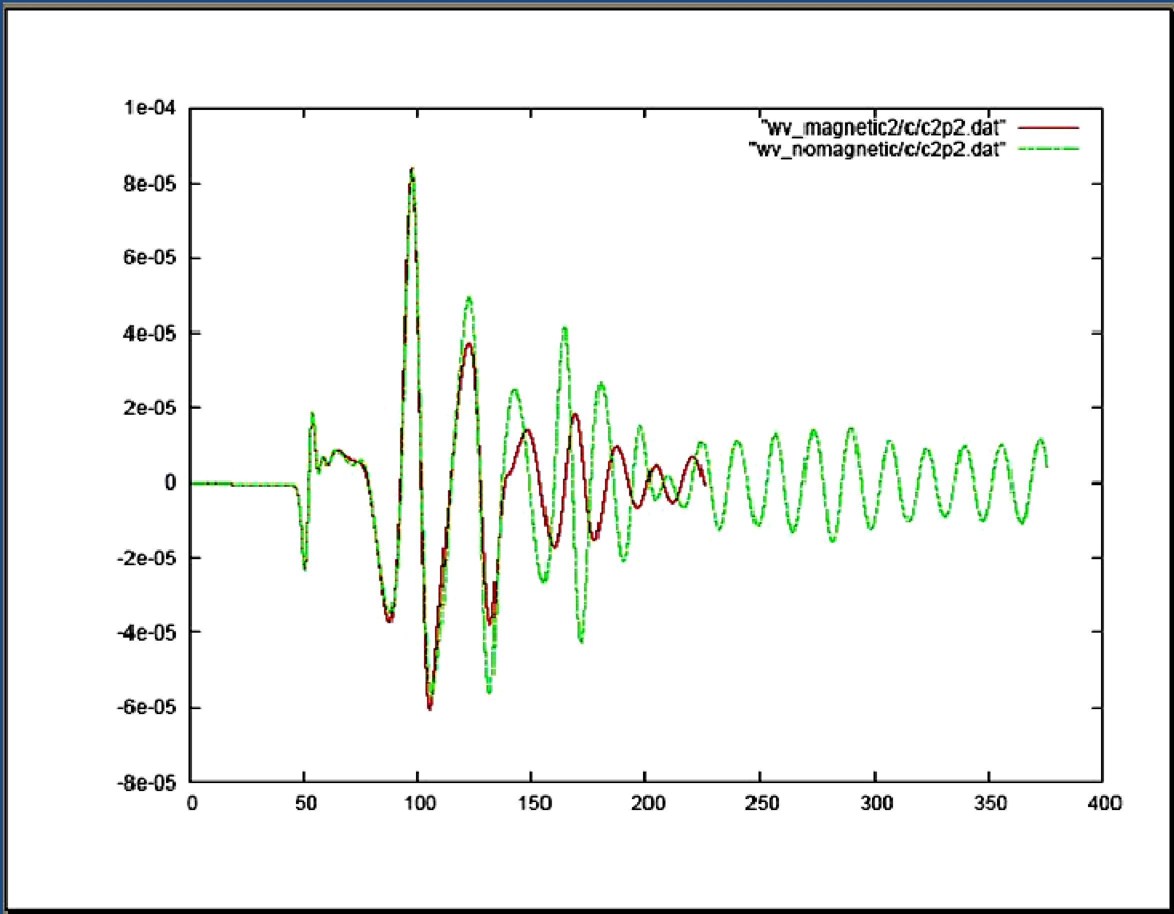
- Needs differential rotation [see Shapiro etal in axisymmetry]
  - Can break the star!
    if sufficiently diff. rot.



On to binaries



Grav waves


"wv_magnetic2/c/c2p2.dat"
"wv_nomagnetic/c/c2p2.dat"
1e-04
8e-05
6e-05
4e-05
2e-05
0
-2e-05
-4e-05
-6e-05
-8e-05
0
50
100
150
200
250
300
350
400

Going up in dims

- Beyond GWs, Astro and fundamental questions in 4D, NR can help in searches for Quant. Grav theories (at a very very very humble level)
  - Take classical limit and analyze what's there
  - Loop quantum gravity → GR in 4D
  - String theory → GR in higher dims coupled with extra fields.

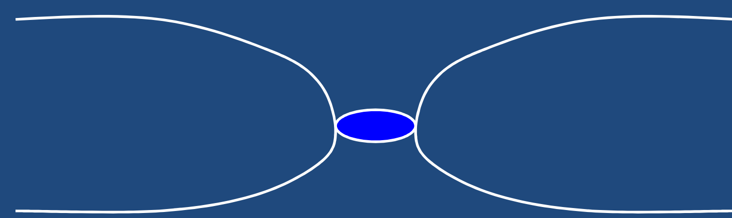
  - Latter case… warning… better knock down lots of dims through symmetry considerations. As in 4D, black hole are handy…

- Features of these black objects?
  - Singularity inside at the classical level (OK)
  - Can not bifurcate unless a naked singularity shows up (OK)
  - Unique in spherical symmetry (NO!)

  - Natural questions!
    - *What are the possible solutions?*
    - *are those stable?*
    - *What's their role (e.g. in black hole information paradox, stability of spacetimes)*

Black strings and bubbles

- Black strings: higher dimensional black holes. In 5D black holes with 'maximum' symmetries are : S^3 hyperspherical black hole or S^2xR cylindrical black hole or black string.



- Bubbles. Topogically 'weird' spacetimes.
  - An initially large sphere can't be shrank to zero size
  - Minkowski spacetime shown to be able to 'quantum tunnel' to a bubble spacetime (Witten bubble)

- Studying both systems require numerical simulations of Einstein equations in higher dimensions (5D) but symmetries allow for treating the black string in 2+1 and bubble in 1+1 dimensions.

•Problem: Kaluza-Klein 'bubbles'

–Positive mass thm (Witten) requires existence of certain structure. In 5d this structure need not be there.

1.Are there negative mass configurations possible in the theory?

2. If so, is cosmic censorship valid?

3. as a singularity forms, nucleation of a bubble can avoid formation of a BH, information paradox would not be a problem (Horowitz)

•Answer to 1. Yes, negative mass configurations found

–Witten bubble (82): associated with instability of KK vacuum. More than 1 state with zero total energy.

–Brill-Pfister (89): explicit solutions to 5D vacuum constraints with negative mass.

–Brill-Horowitz (91): generalization to include 'gauge' fields.

•Qn: What's the space-time like?

–Corley-Jacobson (94). Analyze area of the bubble, conclusion: It starts out expanding [collapsing], if this trend continues, unlikely to form a singularity.

•Conjecture: It will keep expanding [collapsing] out (otherwise go through another moment of time symmetry).

•But….. This only from estimates at the initial hypersurface… what does really happen?… Need to solve the eqns…

–Numerical effort (2000). Conclusion: negative mass bubbles expand but not forever…. At some point a naked singularity appears!!! (or does it?)

Revisiting the problem

- Consider, GR in 5d + the option of an Electromagnetic field. Set initial data so that a bubble exists at r_+

$$A_u dx^u = k(r_+^{-n} - r^{-n}) \quad k \in R; n = 2,3....$$

Time symmetry (mom const =0); Hamiltonian constraint

$$U(r) = 1 - m/r + b/r^2 + \tilde{k}^2/r^{2n}$$

With *m,b* constants. In particular M_{ADM}*=m/4…but this* *__can be negative__*

Initial acceleration of the bubble's area [extending Corley-Jacobson]

$$\ddot{A} \propto \left(1 - m - k[n-1][n-2]\right)$$

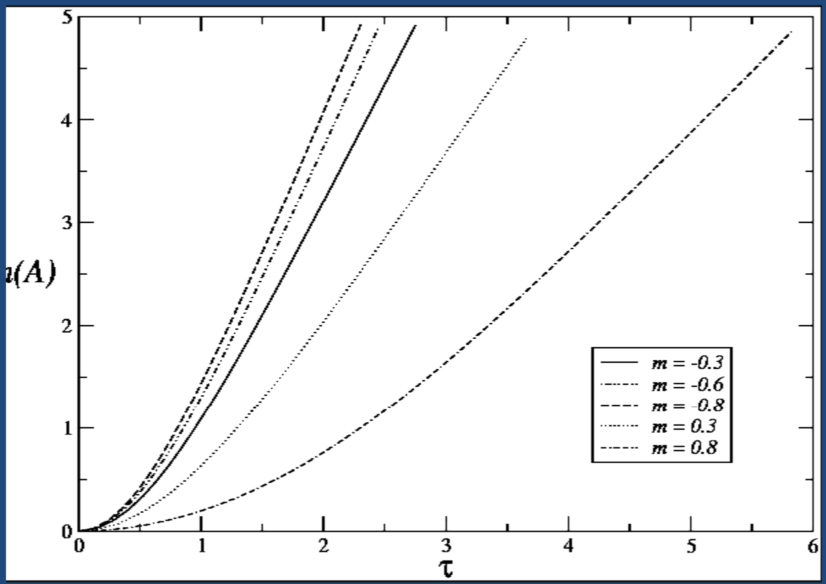
- *n=2. If m<0, bubble expands; m>0 both cases possible*
- *n>2. For k large, arbitrary negative acceleration with negative mass…sounds promising (for something…)!*

Sarbach, LL PRD 03; 05.

Numerical evolution

- Variables functions of *(t,r)* only (1D evolution)
- At outer boundary: constraint preserving boundary conditions [a-la Calabrese,LL,Tiglio 02]
- At bubble, regularity conditions used.

- Proved well posedness at continuum level, translated to the numerical arena thanks to SBP in a first order formulation.
  - Improved resolution at bubble with a non-uniform radial coordinate.



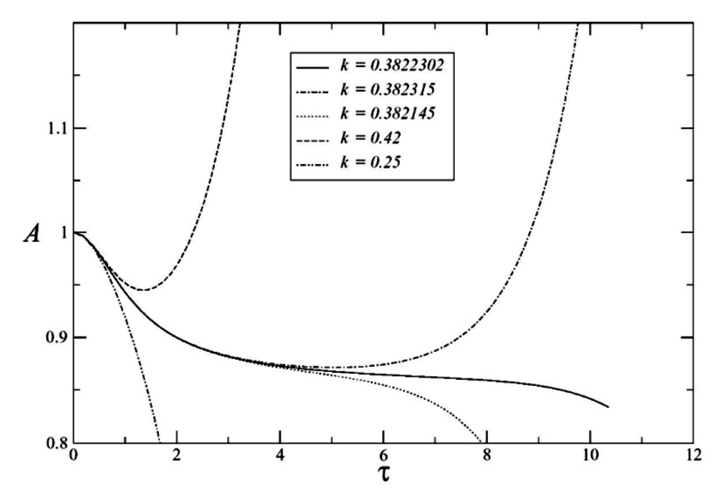
k=0

Case studied numerically previously, no naked singularity found, m<0 expands even faster than m>0

More than we asked for…

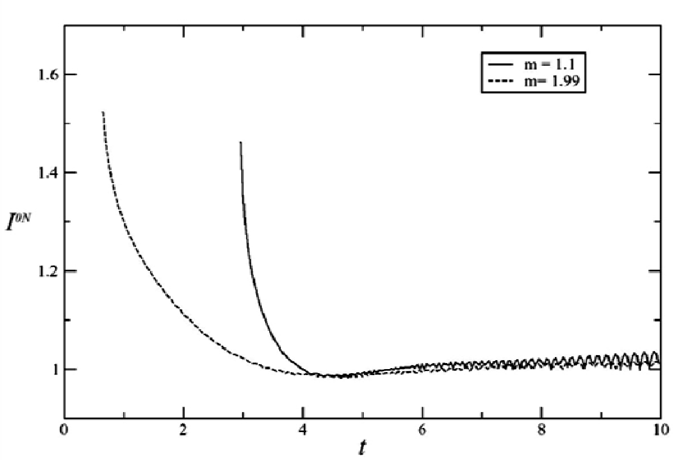
What happens with a non-zero gauge field?

• Choose *n=2*, and stick to cases where bubble starts out collapsing (positive mass)



Depending on field strength, the bubble either collapses ($k<k^*$) *to a black string* or bounces back to expand ($k>k^*$).

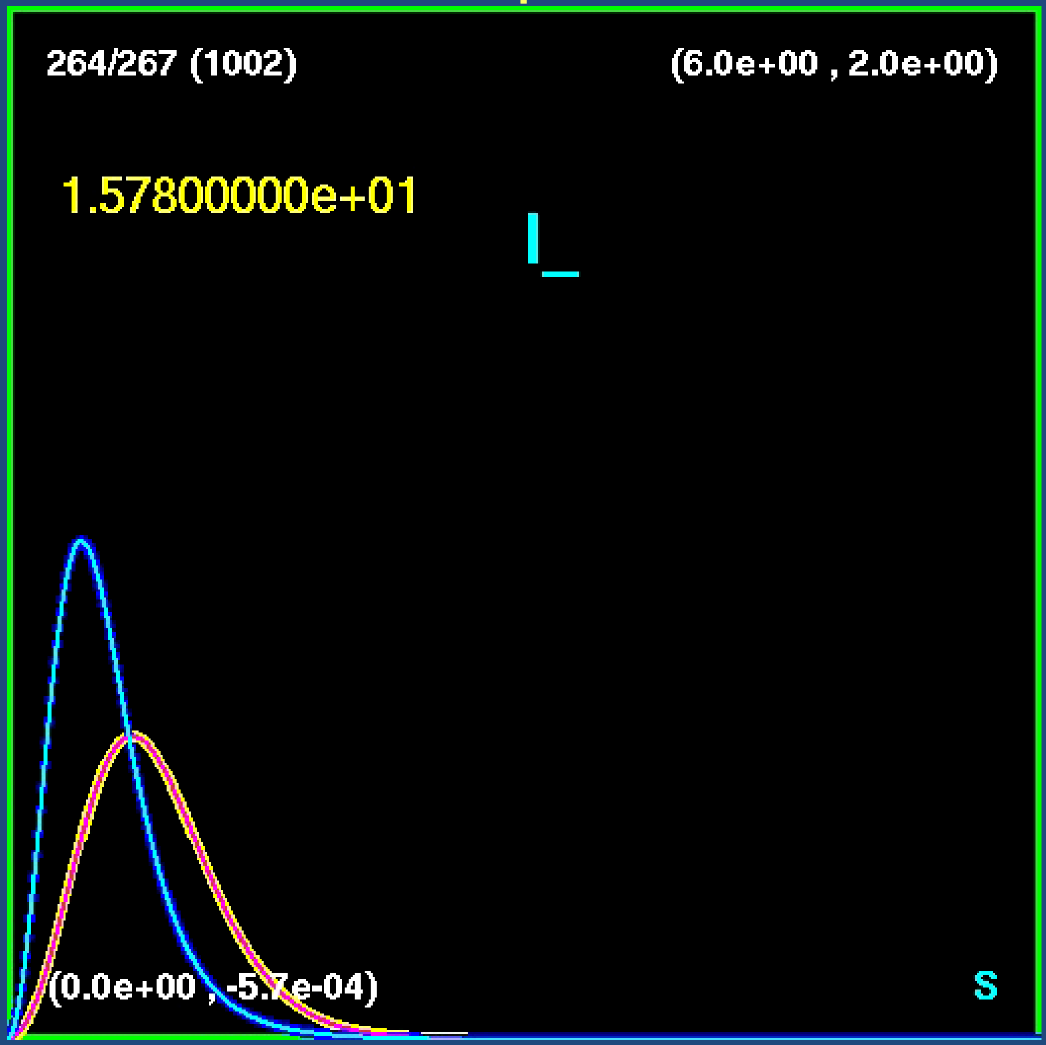
$$\Delta T \xrightarrow[k \to k^*]{} \infty$$



Changes behavior almost always without going through another moment of time symmetry

Last… it appears to approach a stationary solution… does it exist?

Curvature invariant, sub/supra critical behavior



Observation… there must be a static solution at the threshold

- Put static anzats, solve resulting constraint and…

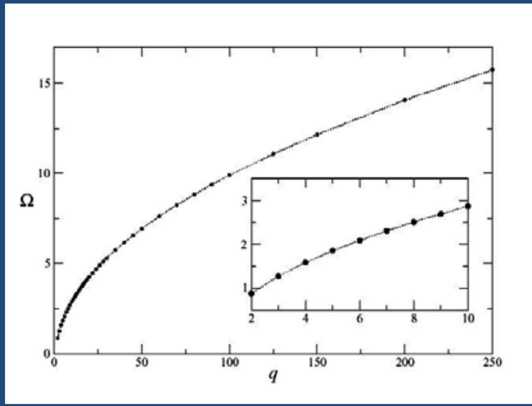
$$ds^2 = -V(r)dt^2 + V(r)/U(r)dr^2 + U(r)/V(r)^2 dz^2 + r^2V(r)dz^2$$

$$A_a dx^a = \pm\frac{1}{2}\sqrt{3(r_+/r_- - 1)}dz/V(r)$$

With $V=(1-r_-/r)$; $U=(1-r_+/r)$. And the parameters are obtained from

$P=4\pi r_+(1-r_-/r_+)^{(3/2)}$ and $M=r_+/4$.

- New solution?… nah… obtained by 'just' analytically continuing that of a charged black string….[found in Horowitz-Maeda 03]
  - Obtained single growing mode



  - Work analytical continuation 'backwards'. Used to show a family of charged black strings becomes *more unstable* as charged is added (opposite to what was conjectured)

- What happened with the negative mass data that started contracting with arbitrary negative acceleration?
  - Bubble shrinks to arbitrarily small sizes, but ‘bounces’ back… cosmic censorship stood its ground, even in such weird scenario…