Beyond binary black holes in numerical relativity

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For movies go to: http://had.liu.edu

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Questions posed to GR people (on the theory side)

- Compact objects: Gravitational waves [pragmatic]
- Compact objects: Role of GR for strongly gravitating/highly dynamical scenarios [pragmatic,fundamental]
- Asymptotic considerations (asymptotic flatness; cosmic censorship) [fundamental]
- 'local' considerations (singularity behavior, clues for/from quantum gravity) [fundamental, pragmatic?]
- 'curious' non-linear theory, no shocks but singularities (makes sense to ask global questions!) [fundamental]

NR…why

If we think hard enough we won't need a computer

With the right resources & techniques we can simulate situations we can't even begin to think through, and thereby provide us with ***completely new and unexpected*** *things to think about*

- Support?
  - Critical Phenomena in GR; Toroidal E.H.; Cauchy Horizons in charged/rot BH's. etc.
- Goal: *Numerical construction of spacetimes.*
  - Access strong field/highly dynamical scenarios
    - Singularity structure
    - Collapse
    - Understand strong gravity's role within astrophysical phenomena
  - Global description
    - Asymptotic behavior
- Added bonus
  - 'Practical' info for GW detectors
  - Key ingredient in GW astronomy

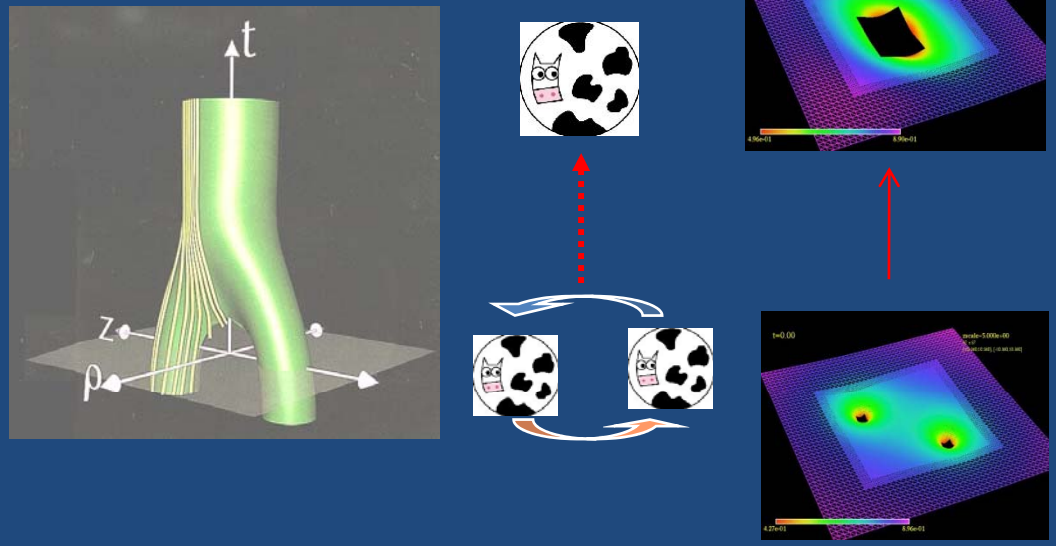
What's not in this talk?

Numerical Relativity in:

- Approaches to the singularity
- Cosmology

Haven't been involved in these (but who knows…)

- Binary bhs



Covered in H. Pfeiffer's talk (though there will some connections)

What's in it then?

- NR for Astrophysics/GW reasons
  - NS-NS & other binaries…
- NR for fundamental qns
  - A bit of critical phenomena
  - Excursions into higher D's worlds

Other interesting systems. GWs and/or Astro connection

- Compact binaries (BH-NS, NS-NS, LXMBs ..etc)
- Gamma ray bursts
- Active galactic nuclei
- Supernovae

Gravity waves but a lot more to learn

Adv LIGO design?

BUT…. Much more beyond GR is needed

- How to represent stars?
- Need to incorporate other physics: magnetic field effects, radiation transport, etc.
  - Each with its own peculiarities (shocks, 6 dimensions, etc)

Examining other binaries in GR

- GWs from binary black holes in remarkable agreement with Post Newtonian results.
  - No surprises, beyond kicks which can be understood in simple terms
  - Problem dependent on few parameters. Final soln described by just 2.
  - PN does not account for internal structure of objects
  - What to expect if not dealing with black holes?
  - How far can PN (& related) approaches?
- Understand role of GR in less ‘clean’ cases
- ‘feed-back’ to binary black holes & kicks prospects

1st binary case

- Boson stars. Compact objects from a complex scalar field [Kaup,Bonnazolla-Ruffini]

$$T_{ab} = (\nabla_{(a}\phi\nabla_{b)}\bar{\phi} - 1/2\, g_{ab}[g^{cd}\nabla_c\phi\nabla_d\bar{\phi} + m^2(|\phi|^2)/2])$$

- Resulting 'stars' share features with TOV stars..
  - Stable stars remain coherent
  - Unstable stars either collapse to a BH or disperse away.
  - Yet… do not yield shocks or contact discontinuities, nor do they have singularities lurking in them.
- Interest:
  - *Mathematical*: solitonic behavior similar to Q-balls
  - *Cosmological*: supermassive objects at centers of galaxies? Dark matter candidate?
  - *Astrophysical*: similar to neutron stars, has stable/unstable branches. Often used as proxies for unknown compact objects

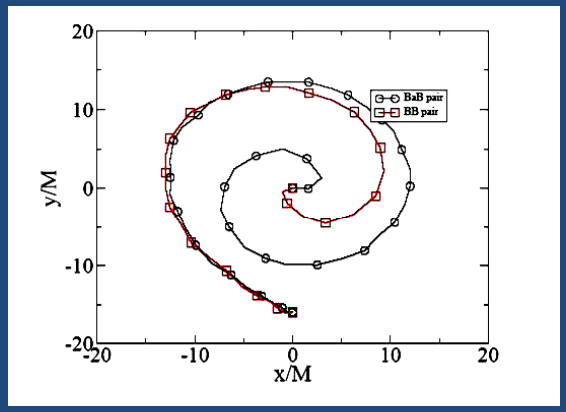
Additional freedom BS → $(BS)^* e^{i\kappa}$

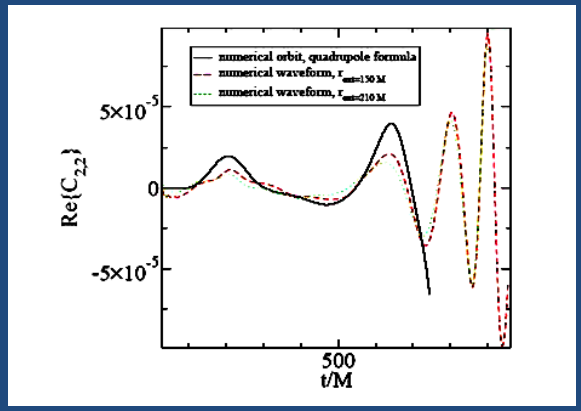
- Simplistic analysis

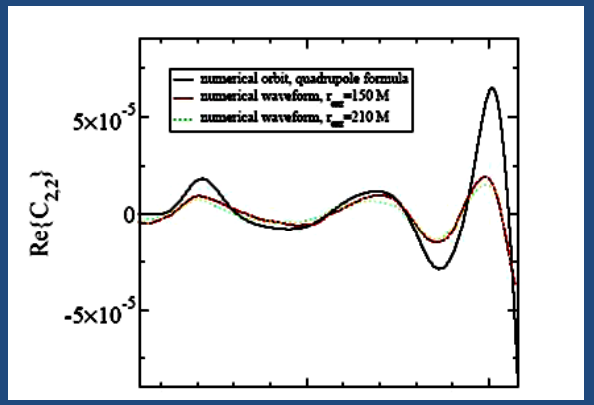
$\rho \sim \rho_1 + \rho_2$ + K cos([1-e] [wt] + κ)

3 cases of 'interaction term':

- Δ (boson – boson; $\kappa = 0$) Movie1; Movie2
- $-\Delta$ (boson – PO boson; $\kappa = \pi$) Movie3
- Δ cos(2wt) (boson – antiboson*; $\kappa = 0$) Movie4

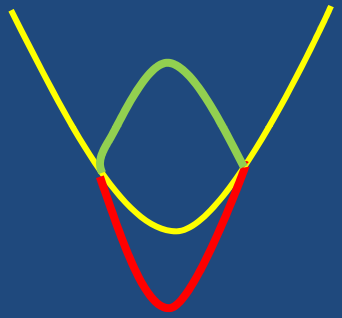






[Palenzuela,L.L.,Liebling 07]

- Binary boson stars, summary
  - Grav Waves. Pattern consistent with PN approx as long as internal effects and/or a BH forms
  - Qualitative behavior can be understood in simple terms



  - Curious behavior:
    - Angular momentum is quantized J = n Q. Set up such:
      - total mass > critical mass for BH formation
      - Total angular momentum > Q but also $J/M^2>1$.
      - Final outcome always a BH!

BUT…. How much do you believe in boson stars?

Binaries with `neutron stars'?

- Now, must deal with 2 types of eqns:
  - Linearly degenerate: Einstein equations
  - Truly non-linear: Hydrodynamic eqns

  Must mix-and-match two different techniques.
  - Also…. NS known to have magnetic fields.
  - Can have strong effects in the dynamics
    - E.g. MRI instability can significantly affect multipolar structure.

$$T_{ab}=(\rho_o(1+\varepsilon) + P)\, u_a\, u_b + P\, g_{ab} + F_{ac}\, F^c{}_b - \tfrac{1}{4}\, g_{ab}\, F_{cd}\, F^{cd}$$

$$J_a + (J^b\, u_b)\, u_a = \sigma\, F_{ab}\, u^b$$

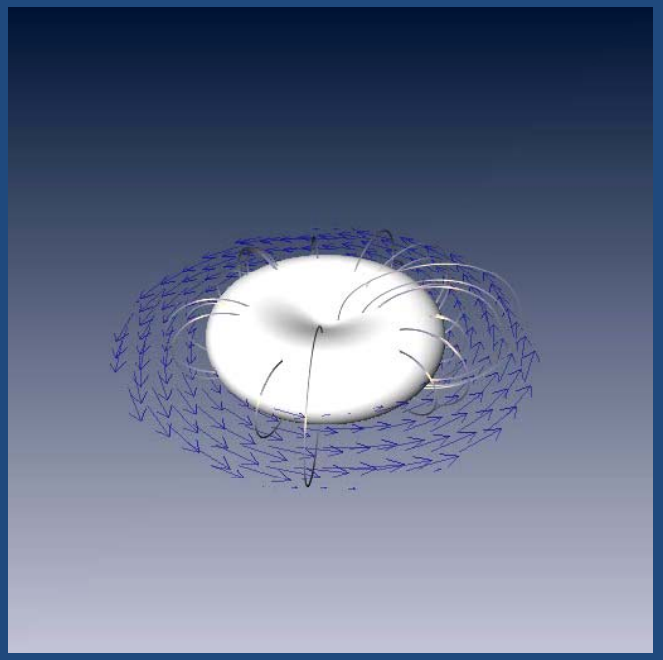
(but $\rightarrow \sigma$ infinty)

[Anderson,Hirschmann,LL,Liebling,Motl,Palenzuela,Tohline, 07]

Dynamics not simple anymore

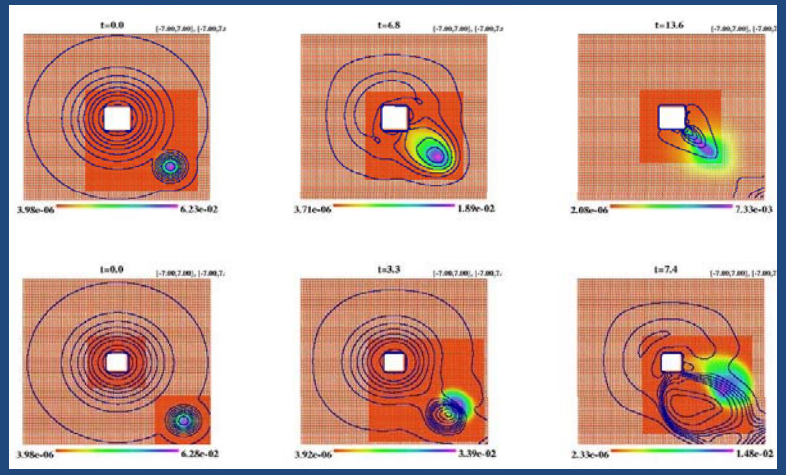
- ID. Differentially rotating neutron star
  - No magnetic field, star preserves its shape

- Magnetic field seeded in through truncation error perturbation.
  - Instability takes off

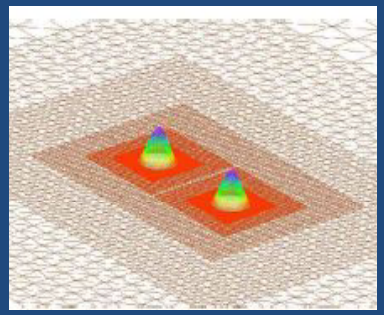


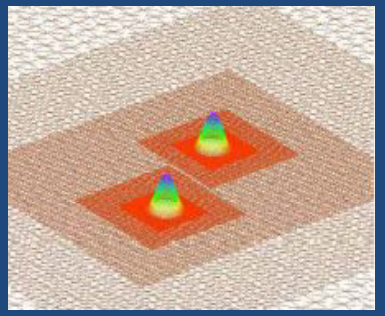
The ‘other’ binaries for GWs

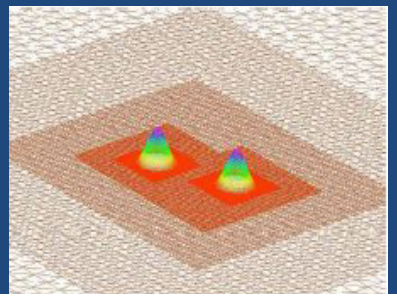
- BH-NS
  - Outcome: BH + disk
  - *Eqn of state* determines if/when stars breaks appart.
  - GW will carry this info.
  - Key for short gamma ray burst models
- NS-NS
  - Outcome: BH + disk; hypermassive star?
  - *Eqn of state* determines dynamics
  - GW carries this info
  - Key for short gamma ray burst models



[Shibata-Uryu,Duez-Kidder-Teukolsky Palenzuela-LL,,etal]



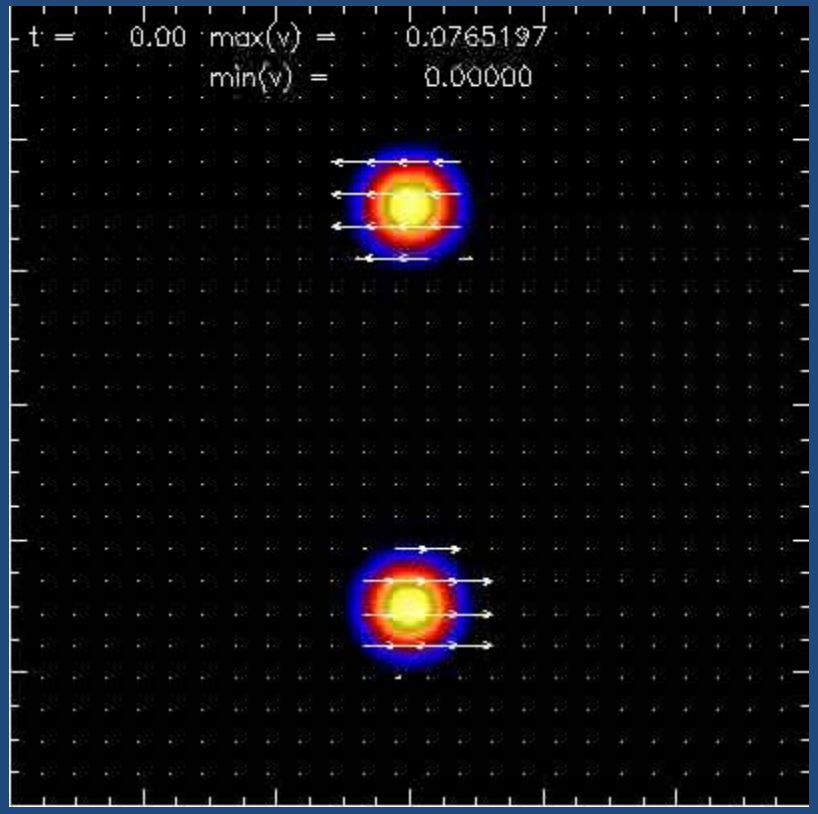




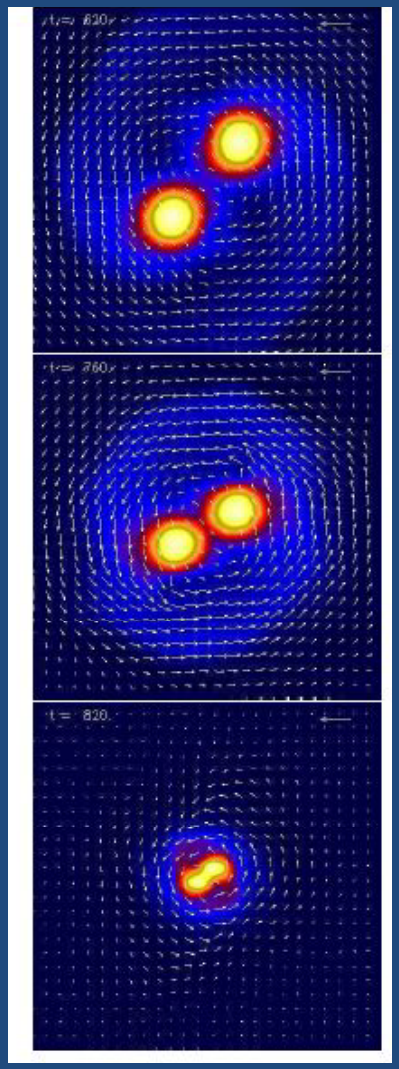
BNS: Setup

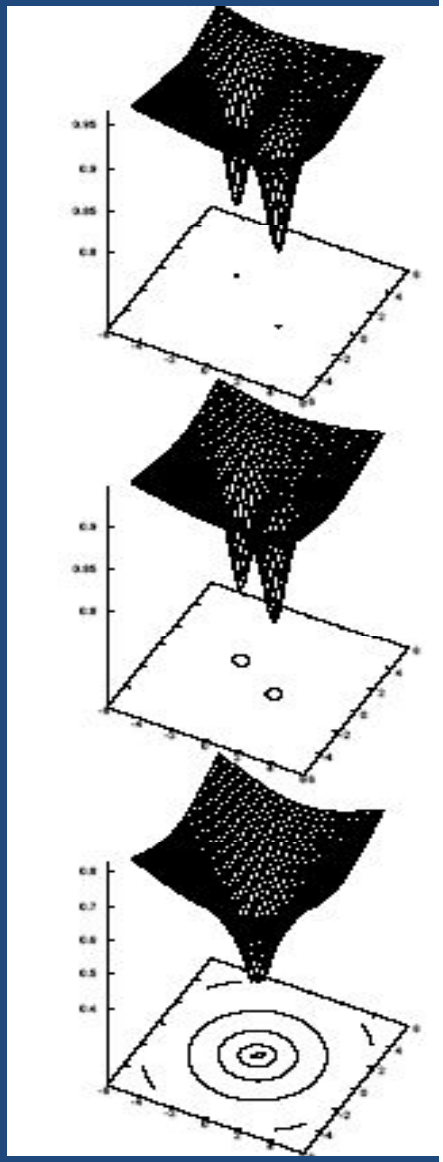
- 2 TOV stars. Compaction ration ~ 0.1
- Eccentric orbit
- Domain: $[-120R_s,120R_s]^3$ (usually < $[-8R_s,8R_s]$ and exploiting symmetries)
- Grav. Waves measured at 30-70R_s (usually $6R_s$)
- Resolution up to 40 pts across each direction of star. Adaptive mesh refinement through shadow hierarchy + some fixed levels out in the boondocks…

Dynamics I (BH outcome)

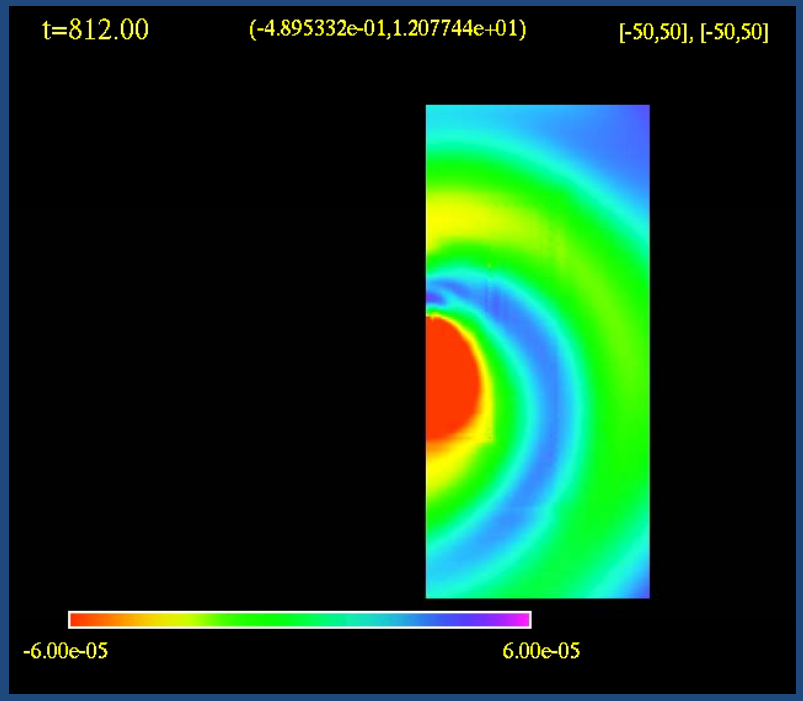


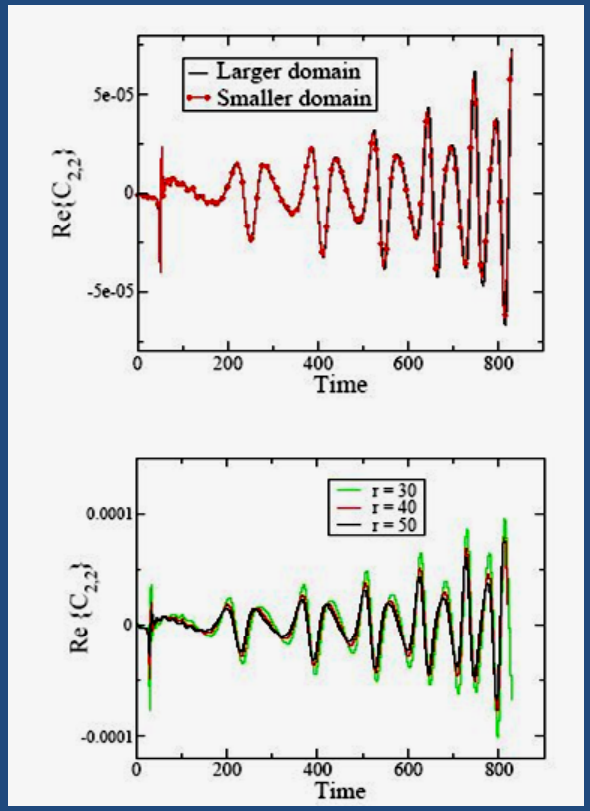
(Shown: 1/20th of total comp. domain)

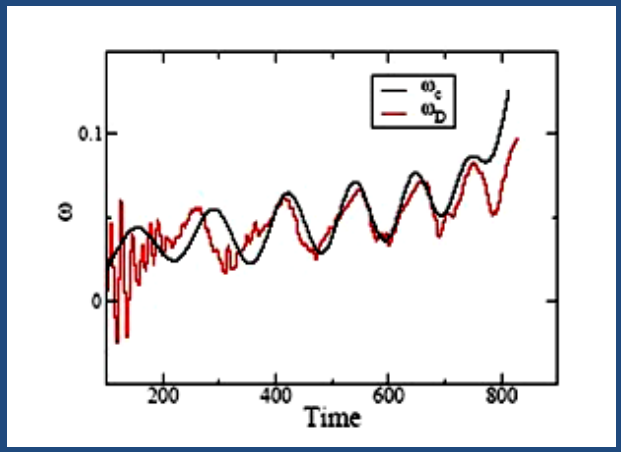




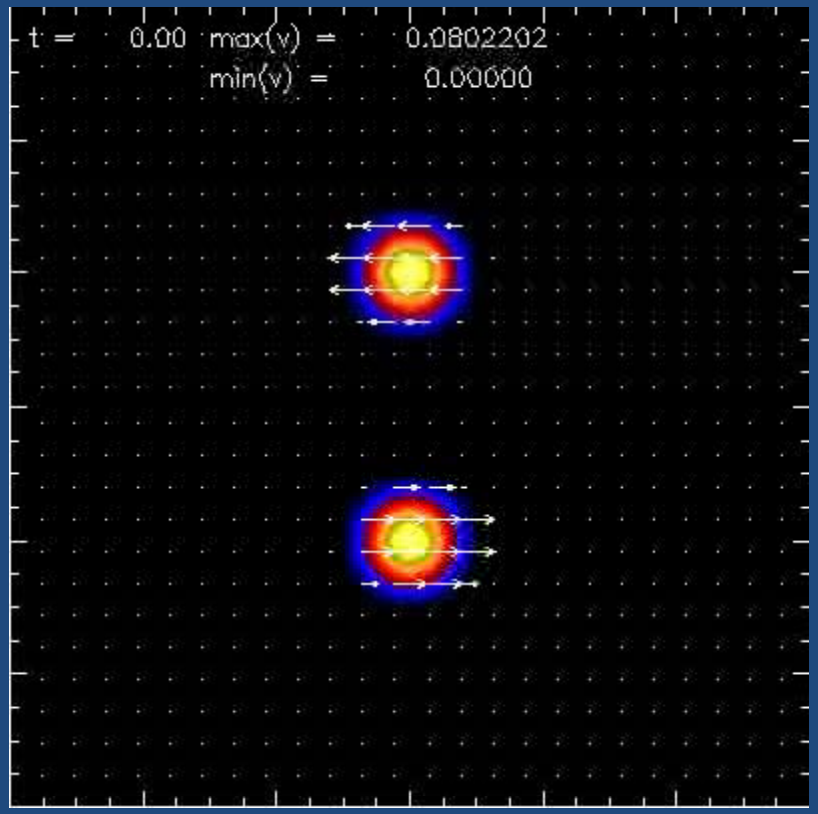
Grav. waves

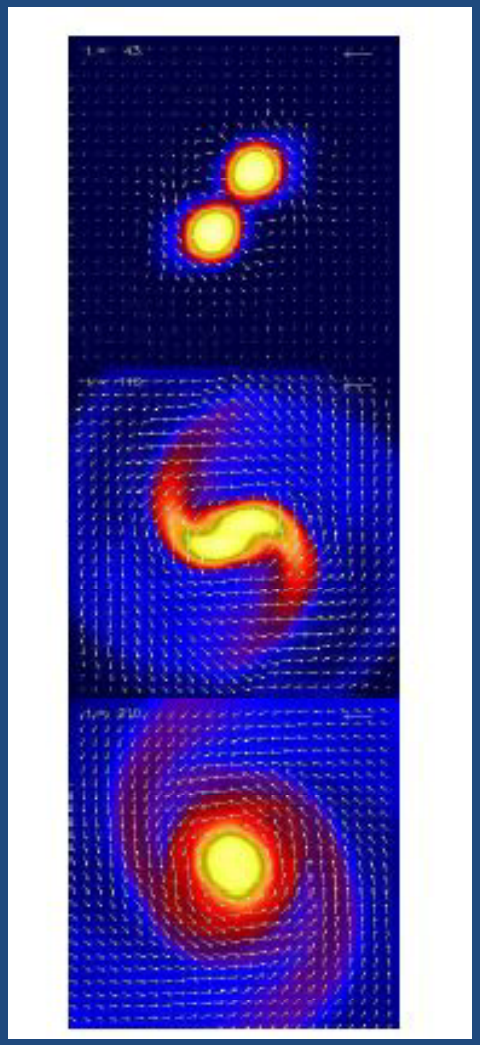

t=812.00
(-4.895332e-01,1.207744e+01)
[-50,50], [-50,50]
-6.00e-05
6.00e-05

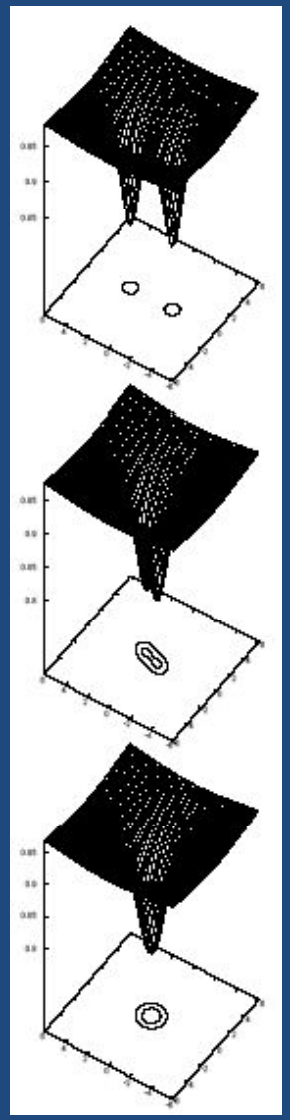

Larger domain
Smaller domain
5e-05
0
-5e-05
Re{C2,2}
0
200
400
600
800
Time
r = 30
r = 40
r = 50
0.0001
0
-0.0001
Re {C2,2}
0
200
400
600
800
Time


ωc
ωD
0.1
0
ω
200
400
600
800
Time

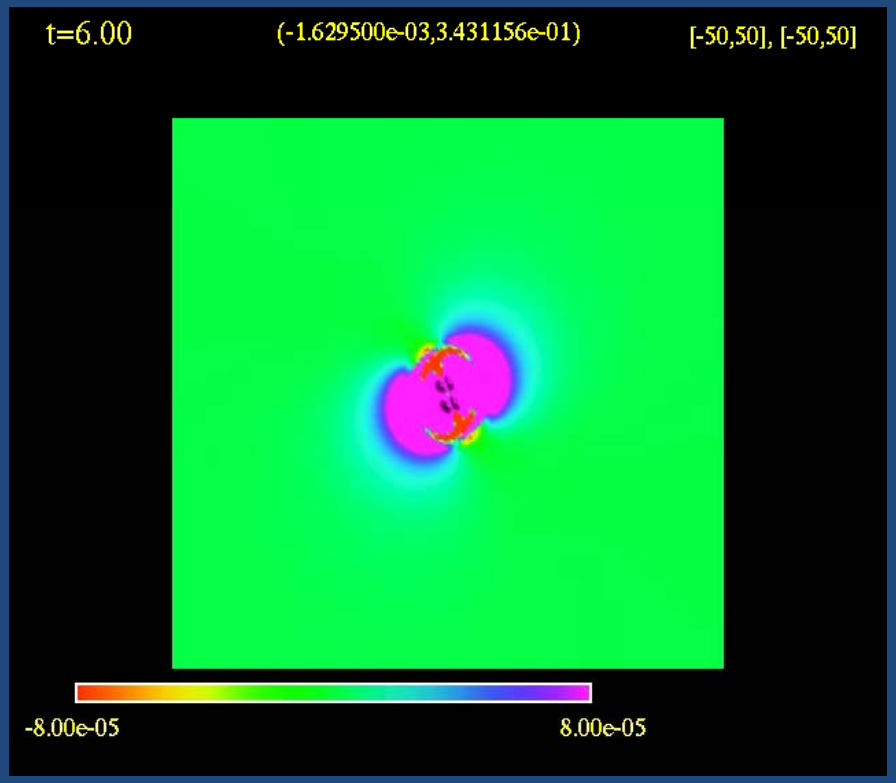
Dynamics II (NS final outcome)







Grav. waves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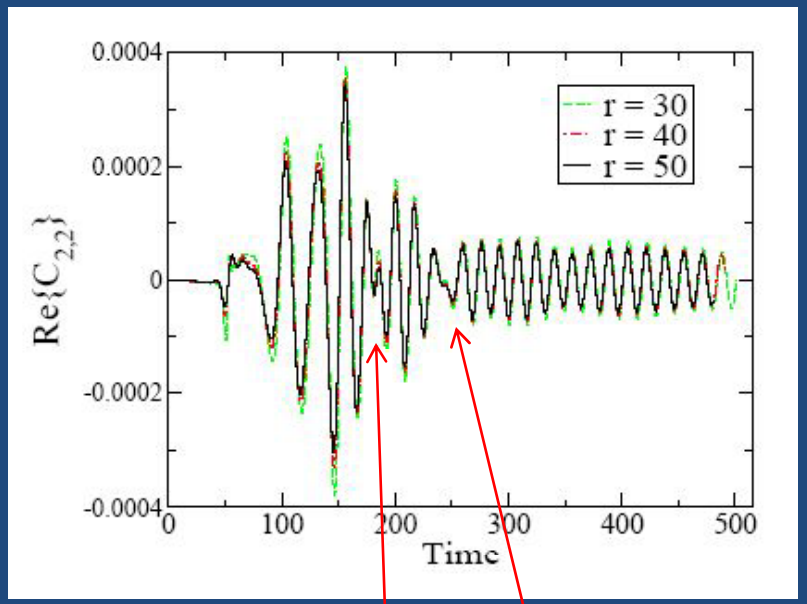
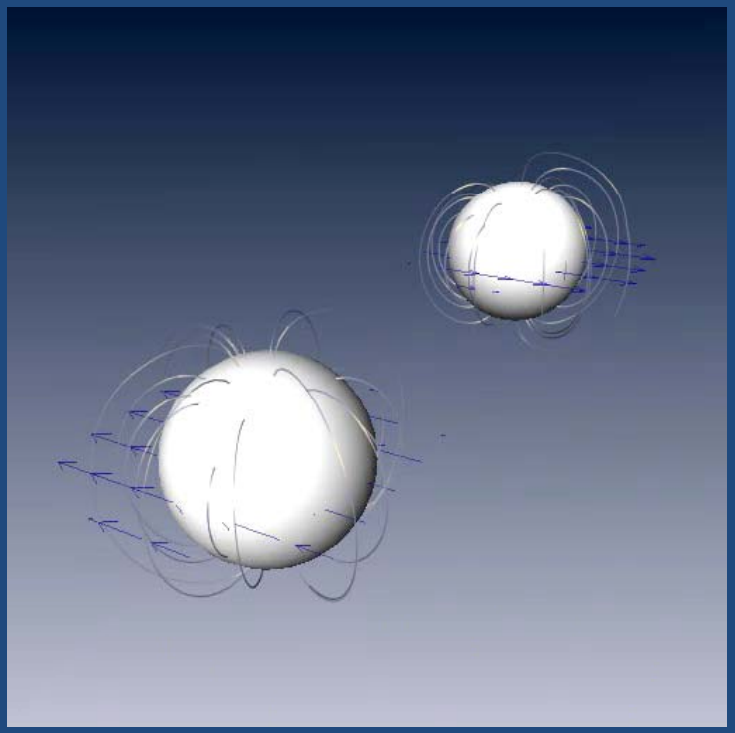
(-1.629500e-03,3.431156e-01)
[-50,50], [-50,50]
-8.00e-05
8.00e-05


0.0004
0.0002
0
-0.0002
-0.0004
0
100
200
300
400
500
Time
Re{C2,2}
r = 30
r = 40
r = 50

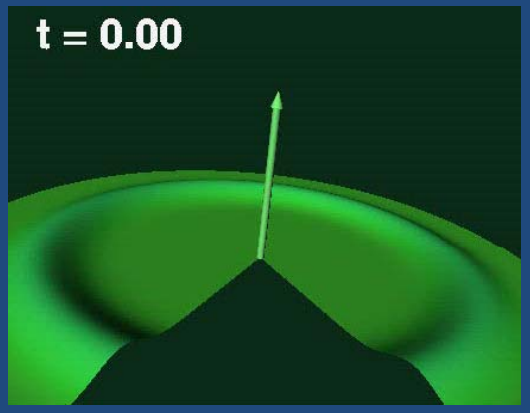
Kicking things up a notch… add B

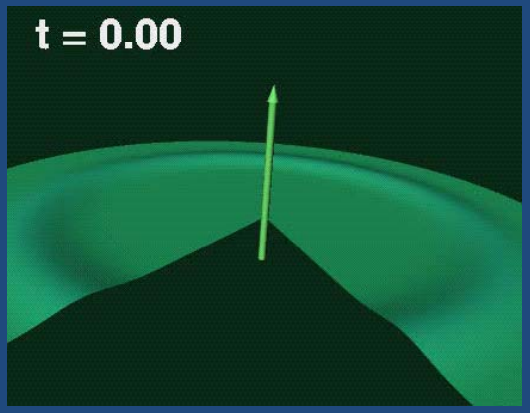


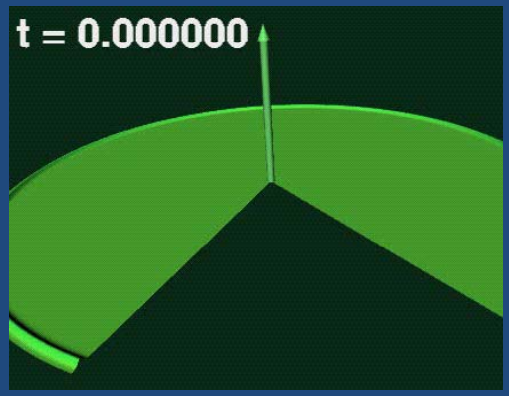
Movie

Enough of GW motivated projects

- Critical phenomena of GR
  - Christodoulou-Kleinerman. ‘weak’ data leaves Minkowski spacetime behind
  - ‘strong’ data collapses to a BH.
  - What’s in the middle?
- Choptuik [….93] .
  - Write an adaptive, robust code to ‘expect the unexpected’.
  - Set-up. Scalar field, tunable parameter.
    - BHs of ‘arbritrarily small mass’ can be formed
    - $M \sim |p-p_x|^k$ with k a universal constant (for the model)
    - But… a lot more! Spacetime ‘repeats’ itself in smaller scales. (self-similarity)



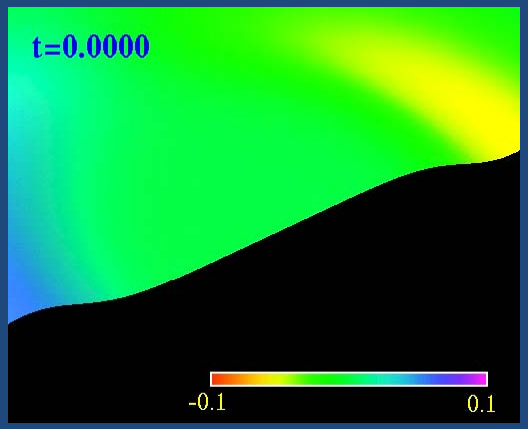




What else do we know?

- Behavior ‘generic’ in spherically symmetric spacetimes
  - 4D with fluids, Yang-Mills, scalar fields (you name it!) [Choptuik-et-al..]
  - >4D with scalar fields [Garfinkle], 5D with gauge fields [Sarbach-LL]
  - More importantly… a posteriori analysis provides a dynamical system description. K ~ Lyapunov coefficient [Gundlach]

- Non-spherical symmetry?
  - Abrahams-Evans. GW collapse
  - Choptuik-Hirschmann-Liebling-Pretorius. Good evidence for the case studied, though critical soln seems to be the copies of spherically symmetric one



Early steps in 3D

- Difficulty tuning beyond:

$$\frac{\left|p^{*}-p\right|}{p^{*}} \approx 1\%$$

- Try studying sub-critical power law scaling of

$$R_{abcd}R^{abcd} \propto \left|p^{*}-p\right|^{-4\gamma}$$

Subcritical Power Law Scaling

- From Fit:

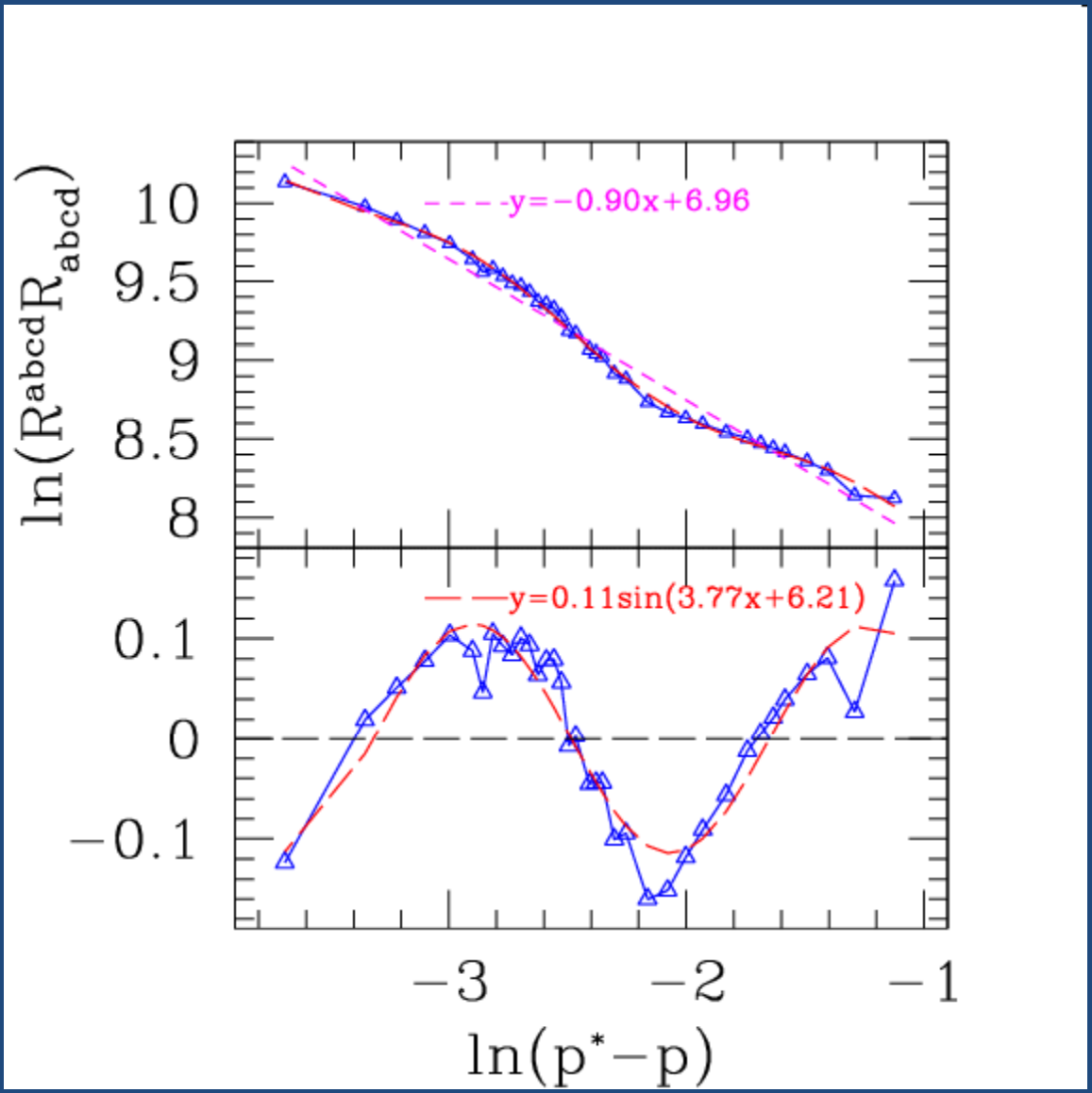
$$\gamma \approx 0.23 \pm ?$$

$$\Delta \approx 0.75 \pm ?$$

- From A&E'93:

$$\gamma = 0.36$$

$$\Delta = 0.60$$



[Liebling et al, in prep]

Going up in dims

- Beyond GWs, Astro and fundamental questions in 4D, NR can help in searches for Quant. Grav theories (at a very very very humble level)
  - Take classical limit and analyze what's there
  - Loop quantum gravity → GR in 4D
  - String theory → GR in higher dims coupled with extra fields.

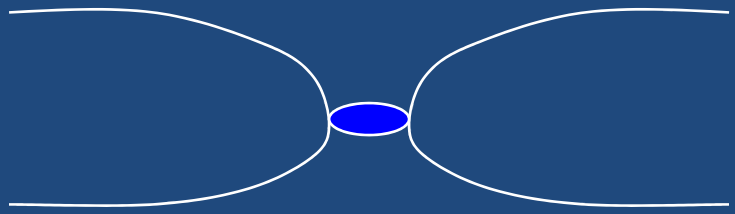
  - Latter case… warning… better knock down lots of dims through symmetry considerations. As in 4D, black hole are handy…

- Features of these black objects?
  - Singularity inside at the classical level (OK)
  - Can not bifurcate unless a naked singularity shows up (OK)
  - Unique in spherical symmetry (NO!)

  - Natural questions!
    - *What are the possible solutions?*
    - *are those stable?*
    - *What's their role (e.g. in black hole information paradox, stability of spacetimes)*

Black strings and bubbles

- Black strings: higher dimensional black holes. In 5D black holes with 'maximum' symmetries are : S^3 hyperspherical black hole or S^2xR cylindrical black hole or black string.

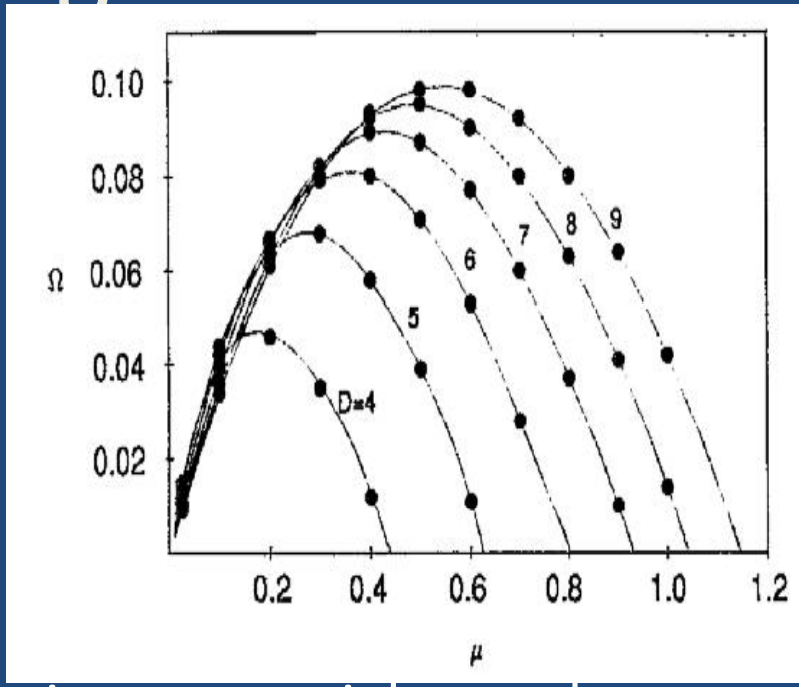


- Bubbles. Topogically 'weird' spacetimes.
  - An initially large sphere can't be shrank to zero size
  - Minkowski spacetime shown to be able to 'quantum tunnel' to a bubble spacetime (Witten bubble)

- Studying both systems require numerical simulations of Einstein equations in higher dimensions (5D) but symmetries allow for treating the black string in 2+1 and bubble in 1+1 dimensions.

Black strings

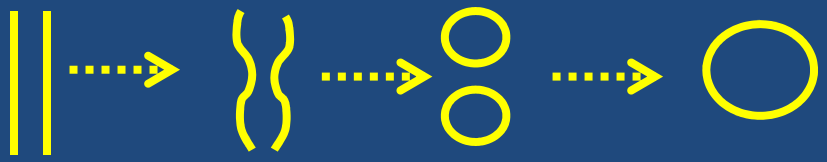
1.- Contain singularities
2.- Ruled by null-rays
3.- Non-unique even in spherical symm



Stability?

- Black string perturbations admit exponential growth for $L > L_c$ (Gregory-Laflamme)
- Entropy $S_{BS} < S_{BH}$ (for a given M)

Conjecture: Black strings will bifurcate

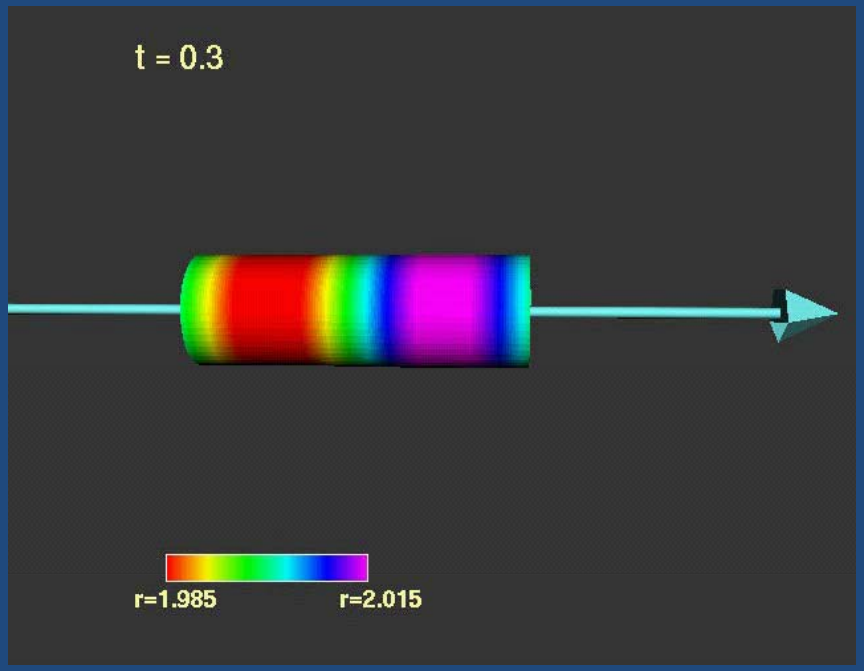


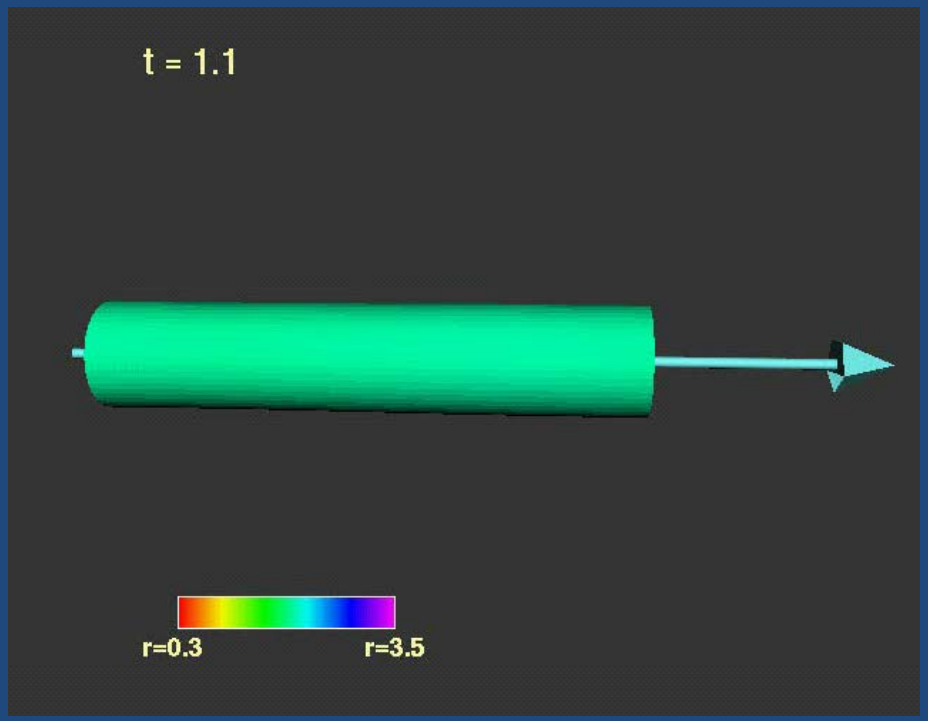
- Conjecture used in many scenarios
  - Density of states from Ads/CFT correspondence
  - Discussions of BH on brane worlds. BH in matrix theory, etc

Somewhat recent developments

- Horowitz-Maeda, can't bifurcate in finite time. *Conjecture: will 'settle' to a non-uniform stationary soln*
- Gubser: transition to soln of first-order type in 5-6D (1^{st}, ~2^{nd} order pert)
- Wiseman: stationary solns which are not the Horowitz-Maeda ones (??)
- Kol: Transition from black string to BH through a conical singularity
- Sorkin-Kol: for high enough dimensions transition is of 2^{nd} order.

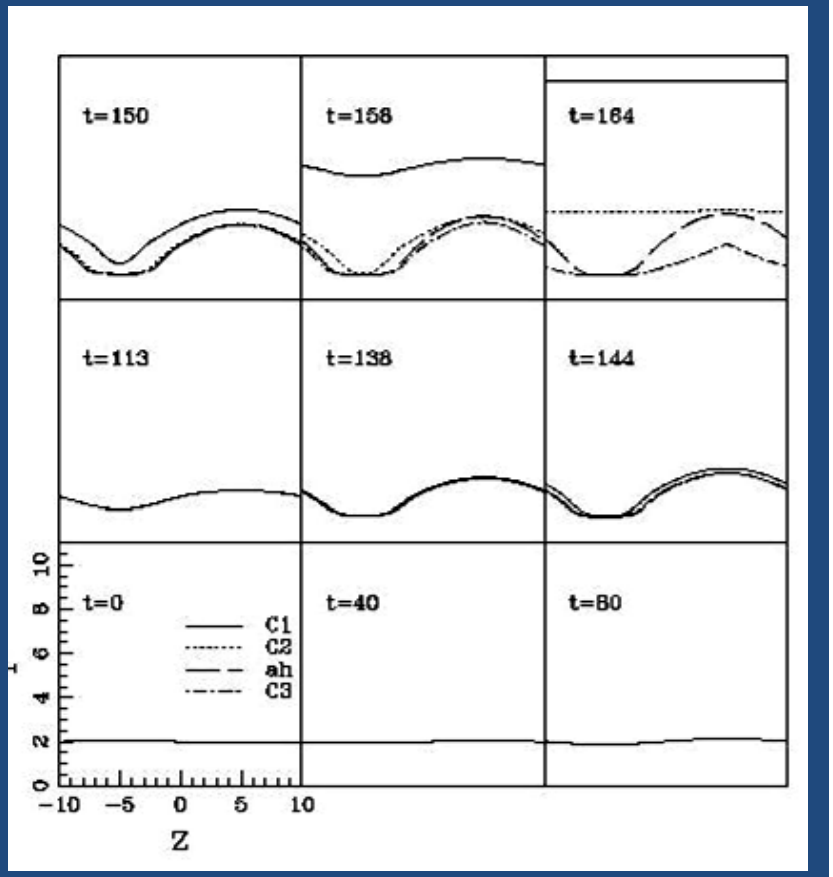
- Qns:
  - *What is the final solution of a perturbed black string?*
  - *Can it bifurcate in 'infinite time'?*
  - *Are Wiseman's solns, physically relevant?*



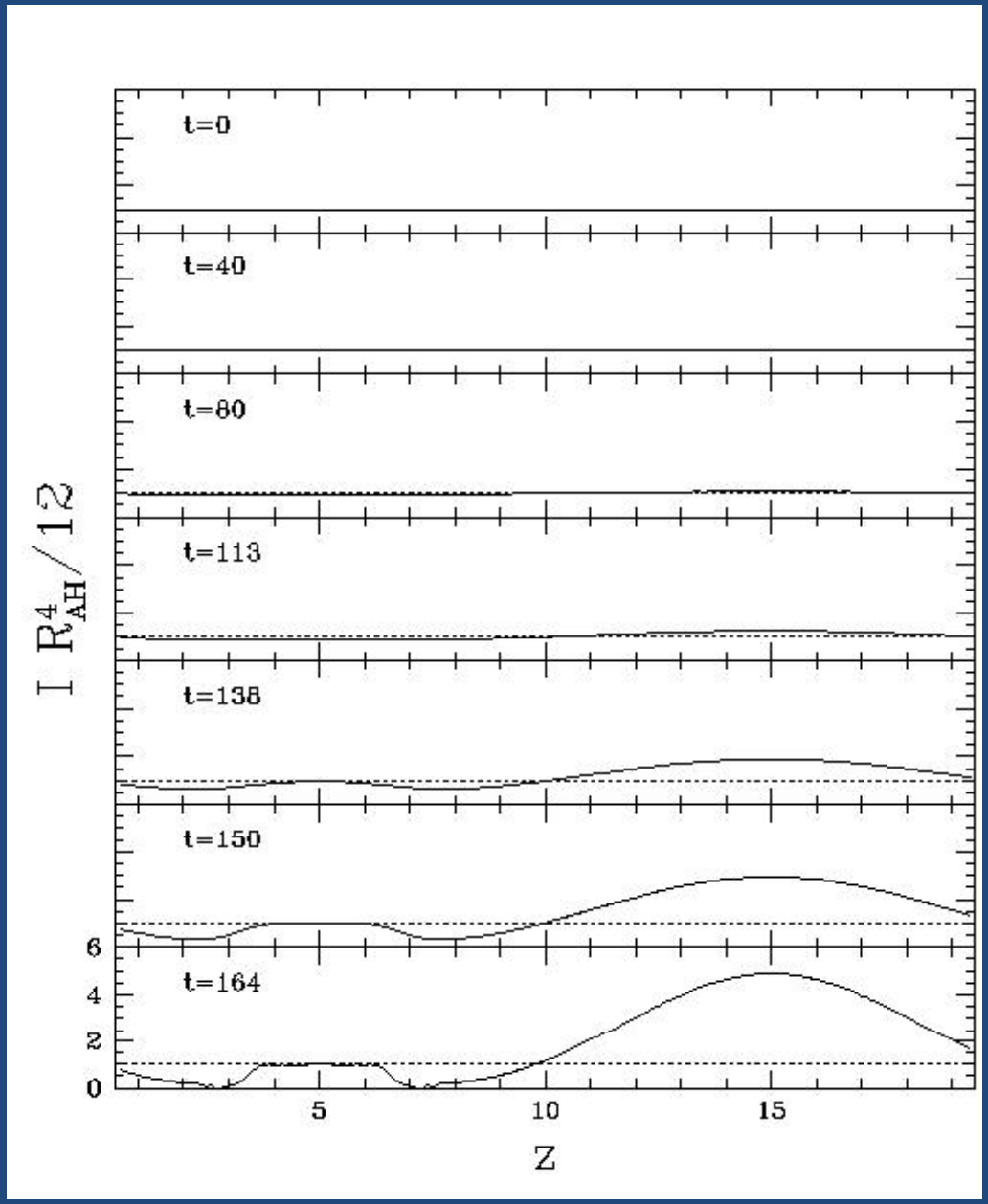


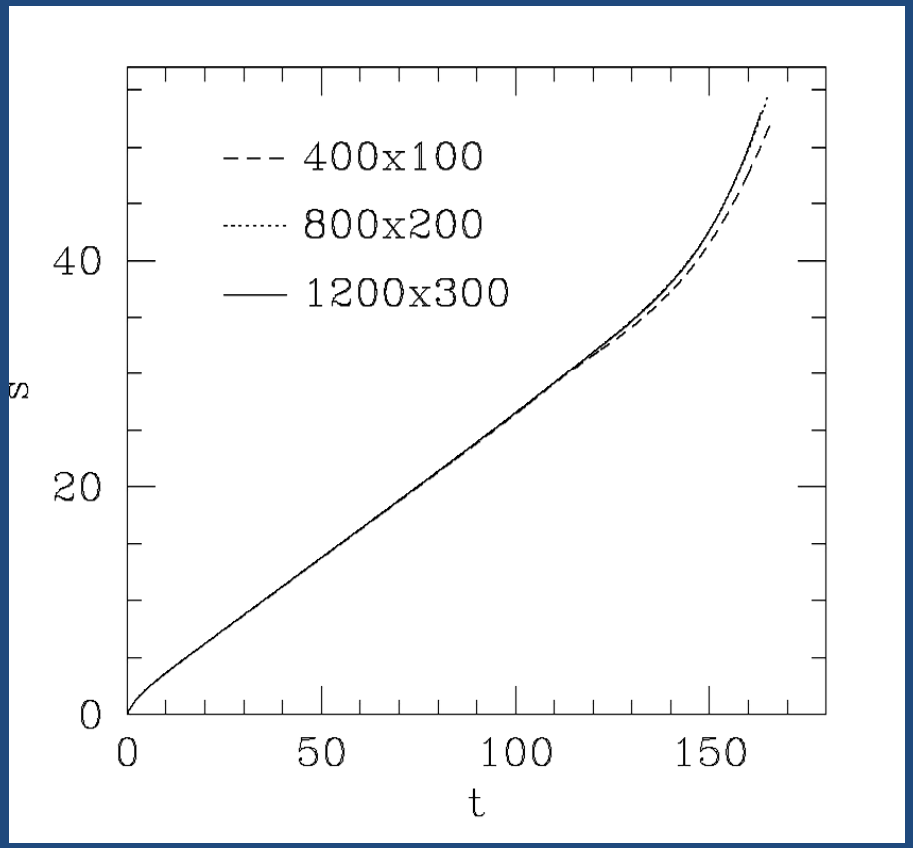
[Choptuik,LL,Pretorius,Olabarrieta,Villegas,Petrik 03]

‘Event’ horizon



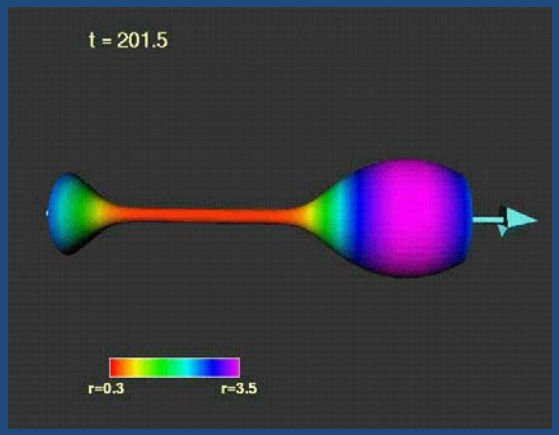
Curvature

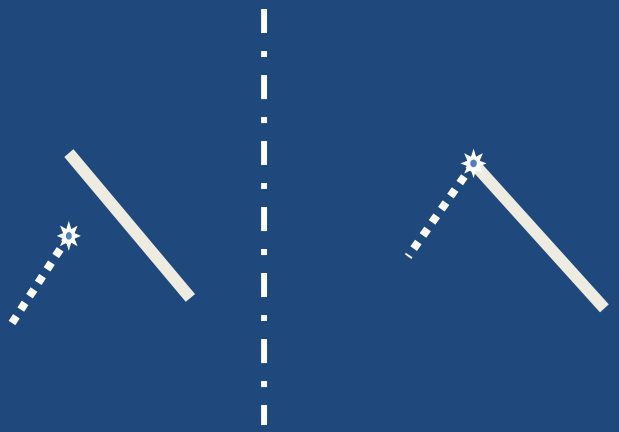




- Affine time, $\lambda = e^s$ growing exponentially ($\sim 10^{22}$)
- "bifurcation" in infinite affine time certainly possible
- 'cascade' of unstable strings also possible

[Garfinkle-LL-Pretorius]





Final words

- NR has a hand on more than BBHs
- Interesting problems in
  - Astrophysics
  - ‘Fundamental’ questions
  - Lots of new playgrounds to come with connections with GR in higher dims (and everything that comes with that)