$$
\begin{aligned}
e &= 1.60217733 \times 10^{-19} C \\
m_e &= 9.109 \times 10^{-31} kg \\
m_n &= 1.675 \times 10^{-27} kg \\
m_p &= 1.673 \times 10^{-27} kg \\
c &= 2.998 \times 10^{8} m/s \\
h &= 6.626 \times 10^{-34} J \cdot s \\
\hbar &= 1.055 \times 10^{-34} J \cdot s \\
R &= 1.0973732 \times 10^{7} m^{-1} \\
a_0 &= \frac{\hbar^2}{m_e k e^2} = 0.529 \mathring{A} = 0.0529 nm \\
H_0 &= 23 \times 10^{-6} \left(km/s\right)/lightyear \\
\Delta t &= \gamma \Delta t' \\
L &= L_p/\gamma \\
x' &= \gamma \left(x - vt\right) \\
t' &= \gamma \left(t - \frac{v}{c^2} x\right) \\
\gamma &= \frac{1}{\sqrt{1 - (v^2/c^2)}} \\
(\Delta s)^2 &= (c\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2 \\
\gamma &\approx 1 + \frac{v^2}{2c^2} + \dots \\
u'_x &= \frac{u_x - v}{1 - (u_x v/c^2)} \\
f_{\text{obs}} &= \frac{\sqrt{1 + (v/c)}}{\sqrt{1 - (v/c)}} f_{\text{source}} \\
(\Delta s)^2 &= (c\Delta t)^2 - (\Delta x)^2 \\
\vec{p} &= \gamma m \vec{u} \\
K &= \gamma mc^2 - mc^2 \\
E &= \gamma mc^2 \\
E^2 &= p^2 c^2 + \left(mc^2\right)^2 \\
F_c &= \frac{mv^2}{r} \\
\vec{F} &= q\vec{v} \times \vec{B} \\
E^2 &= p^2 c^2 + m^2 c^4 \\
e_{\text{total}} &= a\sigma T^4 \\
\lambda_{\max} T &= 2.898 \times 10^{-3} m \cdot K \\
u(f,T) &= \frac{8\pi h f^3}{c^3} \left(\frac{1}{e^{hf/k_B T} - 1}\right) \\
E_{\text{resonator}} &= nhf \quad n = 1, 2, 3, \dots \\
u(f,T) &= \bar{E} N(f) df \\
K_{\max} &= \frac{1}{2} m^e v_{\max}^2 = eV_s \\
K_{\max} &= hf - \phi \\
n\lambda &= 2d \sin\theta \quad n = 1, 2, 3, \dots \\
\lambda' - \lambda &= \frac{h}{m_e c} \left(1 - \cos\theta\right)
\end{aligned}
$$

$$
\begin{aligned}
f' &= f\left(1 + \frac{gH}{c^2}\right) \\
f' &= f\left(1 + \frac{GM}{R_S c^2}\right) \\
\frac{e}{m_e} &= \frac{V\theta}{B^2 l d} \\
\frac{1}{\lambda} &= R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \\
E_i - E_f &= hf \\
m_e v r &= n\hbar \\
r_n &= \frac{n^2 \hbar^2}{m_e k e^2} \quad n = 1, 2, 3, \dots \\
E_n = -\frac{ke^2}{2a_0}\left(\frac{1}{n^2}\right) &= -\frac{13.6 eV}{n^2} \quad n = 1, 2, 3, \dots \\
\frac{1}{\lambda} &= \frac{f}{c} = \frac{ke^2}{2a_0 hc}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \\
\lambda &= \frac{h}{p} \\
v_p &= \frac{\omega}{k} \\
v_g &= \left.\frac{d\omega}{dk}\right|_{k_0} \\
v_p &= c\sqrt{1 + \left(\frac{mc}{\hbar k}\right)^2} \\
f(x) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} a(k) e^{ikx} dk \\
a(k) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f(x) e^{-ikx} dx \\
\Delta p_x \Delta x &\geq \frac{\hbar}{2} \\
\Delta E \Delta t &\geq \frac{\hbar}{2} \\
P(x)dx &= |\Psi(x,t)|^2 dx \\
1 &= \int_{-\infty}^{+\infty} |\Psi(x,t)|^2 dx \\
\Psi_k(x,t) &= Ae^{i(kx-\omega t)} = A\left[\cos\left(kx - \omega t\right) + i\sin\left(kx\right.\right. \\
i\hbar \frac{\partial \Psi}{\partial t} &= -\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} + U(x)\Psi \\
E\psi(x) &= -\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + U(x)\psi(x) \\
E_n &= \frac{\hbar^2 k^2}{2m} = \frac{n^2 \pi^2 \hbar^2}{2mL^2} \\
\psi_n(x) &= \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right) \\
\delta &= \frac{1}{\alpha} = \frac{\hbar}{\sqrt{2m\left(U - E\right)}}
\end{aligned}
$$

$$E_n \approx \frac{n^2\pi^2\hbar^2}{2m\left(L+2\delta\right)^2} \quad n=1,2,3,\ldots$$

$$E_n = \left(n+\frac{1}{2}\right)\hbar\omega \quad n=0,1,2,3,\ldots$$

$$<x> = \int_{-\infty}^{+\infty} x|\Psi(x,t)|^2 dx$$

$$<f> = \int_{-\infty}^{+\infty} f(x)|\Psi(x,t)|^2 dx$$

$$\Delta x = \sqrt{<x^2> - <x>^2}$$

$$<p> = \int_{-\infty}^{+\infty} \Psi^* \left(\frac{\hbar}{i}\right)\frac{\partial}{\partial x}\Psi dx$$

$$<Q> = \int_{-\infty}^{+\infty} \Psi^*[Q]\Psi dx$$

$$[H] = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + U(x)$$

$$[E] = i\hbar\frac{\partial}{\partial t}$$

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi + U(\vec{r})\Psi$$

$$E\psi(\vec{r}) = -\frac{\hbar^2}{2m}\nabla^2\psi(\vec{r}) + U(\vec{r})\psi(\vec{r})$$

$$|p_x| = \hbar k_1 = n_1\frac{\pi\hbar}{L} \quad n_1 = 1,2,\ldots$$

$$E = \frac{1}{2m}\left(|p_x|^2 + |p_y|^2 + |p_z|^2\right)$$

$$\psi(\vec{r}) = \psi(r,\theta,\phi) = R(r)\Theta(\theta)\Phi(\phi)$$

$$x = r\sin\theta\cos\phi$$

$$y = r\sin\theta\sin\phi$$

$$z = r\cos\theta$$

$$|\vec{L}| = \sqrt{\ell\left(\ell+1\right)}\hbar \quad \ell = 0,1,2,\ldots$$

$$L_z = m_\ell\hbar \quad m_\ell = 0,\pm1,\ldots,\pm\ell$$

$$\cos\theta = \frac{L_z}{|\vec{L}|}$$

$$\Psi(r,\theta,\phi,t) = R(r)Y_\ell^{m_\ell}(\theta,\phi)e^{-i\omega t}$$

$$E_n = -\frac{ke^2}{2a_0}\left[\frac{Z^2}{n^2}\right] \quad n=1,2,\ldots$$

$$P(r) = r^2|R(r)|^2$$

$$<r> = \int_0^\infty rP(r)dr$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \left(\frac{2}{r}\right)\frac{\partial}{\partial r} + \frac{1}{r^2}\left[\frac{\partial^2}{\partial\theta^2} + \cot\theta\frac{\partial}{\partial\theta} + \csc^2\theta\frac{\partial^2}{\partial\phi^2}\right]$$

$$dV = dx\ dy\ dz = r^2\sin\theta\ d\phi\ d\theta\ dr$$

$$\vec{\mu} = \frac{q}{2m}\vec{L}$$

$$\omega_L = \frac{e}{2m_e}B$$

$$U = -\vec{\mu}\cdot\vec{B}$$

$$E = E_0 + \hbar\omega_L m_\ell$$

$$\vec{\mu_s} = \frac{q}{2m_e}\vec{S}$$

$$S_z = m_s\hbar$$

$$|\vec{S}| = \sqrt{s\left(s+1\right)}\hbar = \frac{\sqrt{3}}{2}\hbar$$

$$\vec{\mu} = \vec{\mu_0} + \vec{\mu_s} = \frac{-e}{2m_e}\left[\vec{L} + g\vec{S}\right]$$

$$\psi(\vec{r_1},\vec{r_2}) = \pm\psi(\vec{r_2},\vec{r_1})$$

$$Z \equiv \frac{\Delta\lambda}{\lambda}$$

$$Z = \sqrt{\frac{c+v}{c-v}} - 1$$

$$v = H_0 R$$

$$H(t) = \frac{\dot{a}(t)}{a(t)}$$