

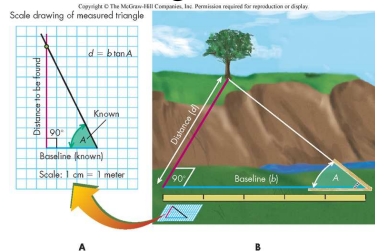
## Chapter 13

### Measuring the Properties of Stars

## The Family of Stars

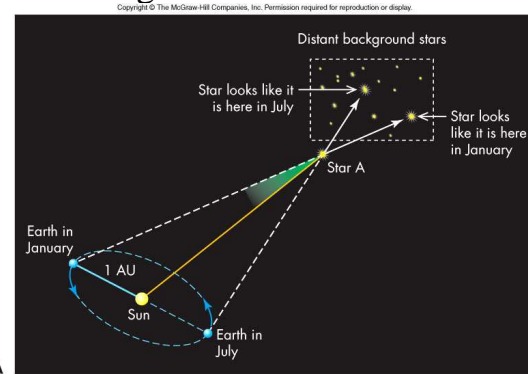
- Those tiny glints of light in the night sky are in reality huge, dazzling balls of gas, many of which are vastly larger and brighter than the Sun
- They look dim because of their vast distances
- Astronomers cannot probe stars directly, and consequently must devise indirect methods to ascertain their intrinsic properties
- Measuring distances to stars and galaxies is not easy
- Distance is very important for determining the intrinsic properties of astronomical objects

## Triangulation

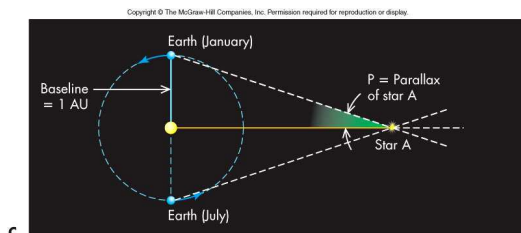


- Fundamental method for measuring distances to nearby stars is **triangulation**:
  - Measure length of a triangle's "baseline" and the angles from the ends of this baseline to a distant object
  - Use trigonometry or a scaled drawing to determine distance to object

## Trigonometric Parallax

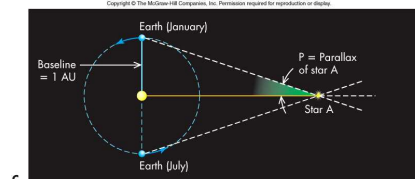


## Calculating Distance Using Parallax



- A method of triangulation used by astronomers is called **parallax**:
  - Baseline is the Earth's orbit radius (1 AU)
  - Angles measured with respect to very distant stars

## Calculating Distance Using Parallax



- The shift of nearby stars is small, so angles are measured in arc seconds
- The parallax angle,  $p$ , is half the angular shift of the nearby star, and its distance in parsecs is given by:

$$d_{pc} = 1/p_{arc\ seconds}$$

- A **parsec** is 3.26 light-years ( $3.09 \times 10^{13}$  km)
- Useful only to distances of about 250 parsecs

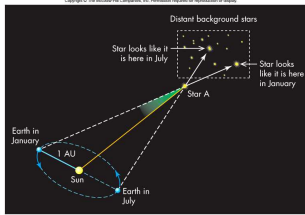
## Example: Distance to Sirius

- Measured parallax angle for Sirius is 0.377 arc second
- From the formula,

$$d_{pc} = 1/0.377$$

$$= 2.65 \text{ parsecs}$$

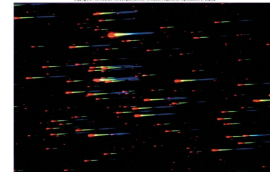
$$= 8.6 \text{ light-years}$$



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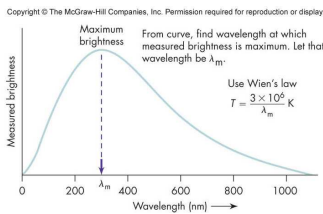
## Light, the Astronomer's Tool

- Astronomers want to know the motions, sizes, colors, and structures of stars
- This information helps to understand the nature of stars as well as their life cycle
- The light from stars received at Earth is all that is available for this analysis



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## Temperature

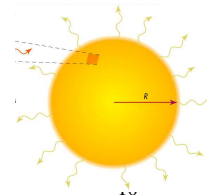
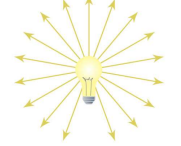


- The color of a star indicates its relative temperature – blue stars are hotter than red stars
- More precisely, a star's surface temperature (in Kelvin) is given by the wavelength in nanometers (nm) at which the star radiates most strongly

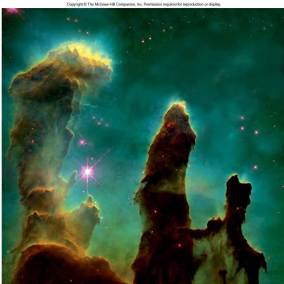
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## Luminosity

- The amount of energy a star emits each second is its **luminosity** (usually abbreviated as  $L$ )
- A typical unit of measurement for luminosity is the watt
- Compare a 100-watt bulb to the Sun's luminosity,  $4 \times 10^{26}$  watts



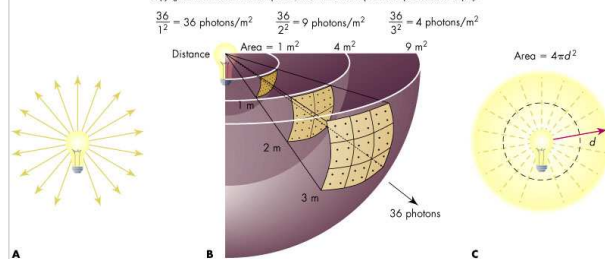
## Luminosity



- Luminosity is a measure of a star's energy production (or hydrogen fuel consumption)
- Knowing a star's luminosity will allow a determination of a star's distance and radius

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## The Inverse-Square Law

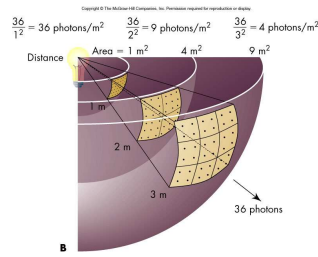


- The **inverse-square law** relates an object's luminosity to its distance and its apparent brightness (how bright it appears to us)

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## The Inverse-Square Law

- This law can be thought of as the result of a fixed number of photons, spreading out evenly in all directions as they leave the source
- The photons have to cross larger and larger concentric spherical shells.
- For a given shell, the number of photons crossing it decreases per unit area



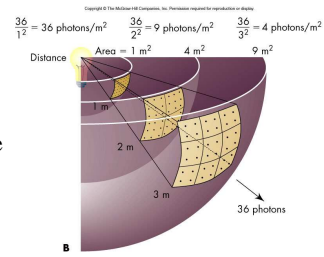
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## The Inverse-Square Law

- The inverse-square law (IS) is:

$$B = \frac{L}{4\pi d^2}$$

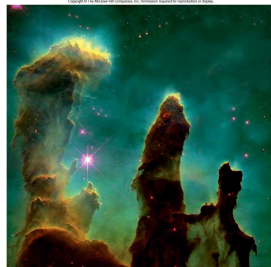
- $B$  is the brightness at a distance  $d$  from a source of luminosity  $L$
- This relationship is called the inverse-square law because the distance appears in the denominator as a square



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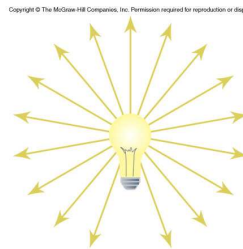
## The Inverse-Square Law

- The inverse-square law is one of the most important mathematical tools available to astronomers:
  - Given  $d$  from parallax measurements, a star's  $L$  can be found (A star's  $B$  can easily be measured by an electronic device, called a photometer, connected to a telescope.)
  - Or if  $L$  is known in advance, a star's distance can be found



$$B = \frac{L}{4\pi d^2}$$

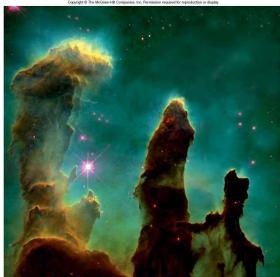
## The “Standard Candle” Method



- If an object's *intrinsic* brightness is known, its distance can be determined from its *observed* brightness
- Astronomers call this method of distance determination the *method of standard candles*
- This method is the principle manner in which astronomers determine distances in the universe

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## Radius



- Common sense: Two objects of the same temperature but different sizes, the larger one radiates more energy than the smaller one
- In stellar terms: a star of larger radius will have a higher luminosity than a smaller star at the same temperature

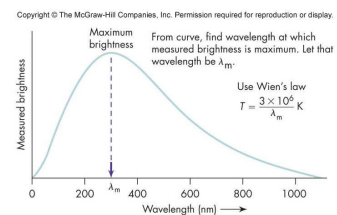
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## Knowing $L$ “In Advance”

- We first need to know how much energy is emitted per unit area of a surface held at a certain temperature
- The Stefan-Boltzmann (SB) Law gives this:

$$B = \sigma T^4$$

- Here  $\sigma$  is the Stefan-Boltzmann constant ( $5.67 \times 10^{-8}$  watts  $\text{m}^{-2}\text{K}^{-4}$ )



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## Tying It All Together

- The Stefan-Boltzmann law only applies to stars, but not hot, low-density gases
- We can combine SB and IS to get:

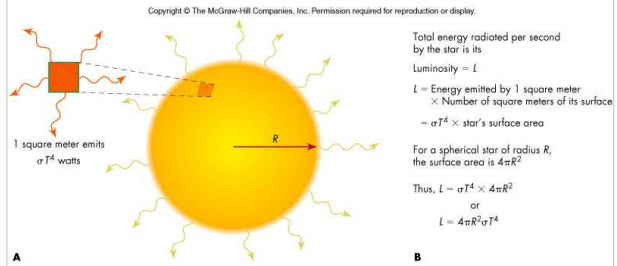
$$L = 4\pi R^2 \sigma T^4$$

- $R$  is the radius of the star
- Given  $L$  and  $T$ , we can then find a star's radius!



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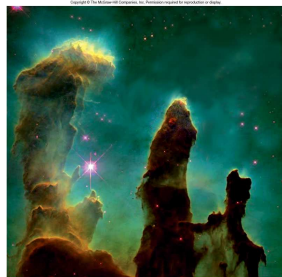
## Tying It All Together



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## Tying It All Together

- The methods using the Stefan-Boltzmann law and interferometer observations show that stars differ enormously in radius
  - Some stars are hundreds of times larger than the Sun and are referred to as *giants*
  - Stars smaller than the giants are called *dwarfs*



$$L = 4\pi R^2 \sigma T^4$$

## The Magnitude Scale

- About 150 B.C., the Greek astronomer Hipparchus measured apparent brightness of stars using units called *magnitudes*
  - Brightest stars had magnitude 1 and dimmest had magnitude 6
  - The system is still used today and units of measurement are called apparent magnitudes to emphasize how bright a star looks to an observer
- A star's apparent magnitude depends on the star's luminosity and distance – a star may appear dim because it is very far away or it does not emit much energy

## The Magnitude Scale

- The apparent magnitude can be confusing
  - Scale runs “backward”: high magnitude = low brightness
  - Modern calibrations of the scale create negative magnitudes
  - Magnitude differences equate to brightness ratios:
    - A difference of 5 magnitudes = a brightness ratio of 100
    - 1 magnitude difference = brightness ratio of  $100^{1/5} = 2.512$

## The Magnitude Scale

- Astronomers use *absolute magnitude* to measure a star's luminosity
  - The absolute magnitude of a star is the apparent magnitude that same star would have at 10 parsecs
  - A comparison of absolute magnitudes is now a comparison of luminosities, no distance dependence
  - An absolute magnitude of 0 approximately equates to a luminosity of  $100L_{\odot}$



## The Spectra of Stars

- A • A star's spectrum typically depicts the energy it emits at each wavelength
- B • A spectrum also can reveal a star's composition, temperature, luminosity, velocity in space, rotation speed, and other properties
- C • On certain occasions, it may reveal mass and radius

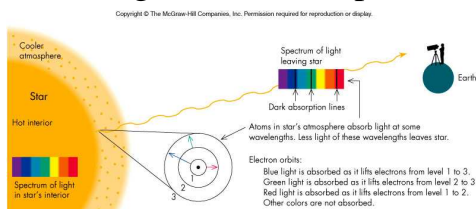
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## Measuring a Star's Composition

- As light moves through the gas of a star's surface layers, atoms absorb radiation at some wavelengths, creating dark absorption lines in the star's spectrum
- Every atom creates its own unique set of absorption lines
- Determining a star's surface composition is then a matter of matching a star's absorption lines to those known for atoms

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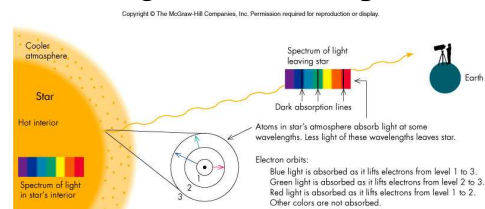
## Measuring a Star's Composition



- To find the quantity of a given atom in the star, we use the darkness of the absorption line
- This technique of determining composition and abundance can be tricky!

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## Measuring a Star's Composition



- Possible overlap of absorption lines from several varieties of atoms being present
- Temperature can also affect how strong (dark) an absorption line is

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## Temperature's Effect on Spectra

- A photon is absorbed when its energy matches the difference between two electron energy levels and an electron occupies the lower energy level
- Higher temperatures, through collisions and energy exchange, will force electrons, on average, to occupy higher electron levels – lower temperatures, lower electron levels

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## Temperature's Effect on Spectra

- Consequently, absorption lines will be present or absent depending on the presence or absence of an electron at the right energy level and this is very much dependent on temperature
- Adjusting for temperature, a star's composition can be found – interestingly, virtually all stars have compositions very similar to the Sun's: 71% H, 27% He, and a 2% mix of the remaining elements

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## Early Classification of Stars

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- Historically, stars were first classified into four groups according to their color (white, yellow, red, and deep red), which were subsequently subdivided into classes using the letters A through N

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## Modern Classification of Stars

- Annie Jump Cannon discovered the classes were more orderly in appearance if rearranged by temperature – Her reordered sequence became O, B, A, F, G, K, M (O being the hottest and M the coolest) and are today known as *spectral classes*



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## Modern Classification of Stars

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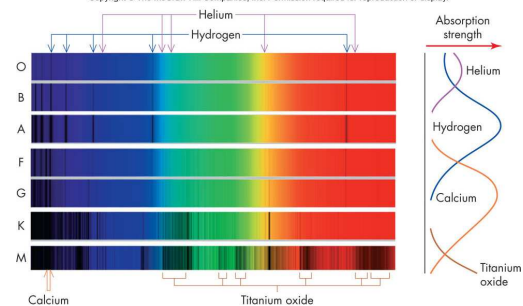


- Cecilia Payne then demonstrated the physical connection between temperature and the resulting absorption lines

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## Modern Classification of Stars

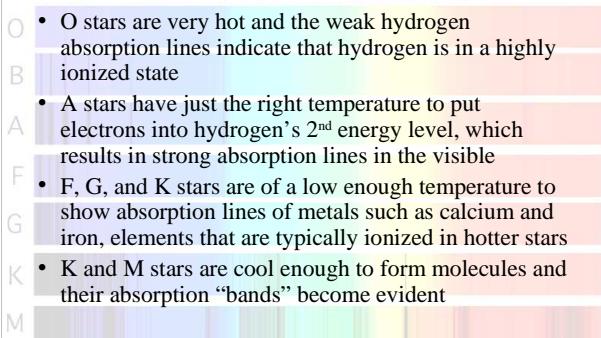
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## Spectral Classification

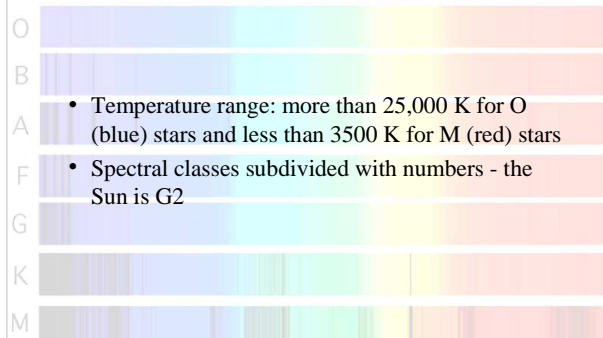
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## Spectral Classification

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## Measuring a Star's Motion

- A star's motion is determined from the Doppler shift of its spectral lines
  - The amount of shift depends on the star's **radial velocity**, which is the star's speed along the line of sight
  - Given that we measure  $\Delta\lambda$ , the shift in wavelength of an absorption line of wavelength  $\lambda$ , the radial speed  $v$  is given by:

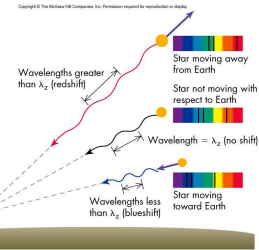
$$v = \left( \frac{\Delta\lambda}{\lambda} \right) c$$

–  $c$  is the speed of light

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## Measuring a Star's Motion

- Note that  $\lambda$  is the wavelength of the absorption line for an object at rest and its value is determined from laboratory measurements on nonmoving sources
- An increase in wavelength means the star is moving away, a decrease means it is approaching – speed across the line on site cannot be determined from Doppler shifts



$$v = \left( \frac{\Delta\lambda}{\lambda} \right) c$$

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## Measuring a Star's Motion

- Doppler measurements and related analysis show:
  - All stars are moving and that those near the Sun share approximately the same direction and speed of revolution (about 200 km/sec) around the center of our galaxy
  - Superimposed on this orbital motion are small random motions of about 20 km/sec
  - In addition to their motion through space, stars spin on their axes and this spin can be measured using the Doppler shift technique – young stars are found to rotate faster than old stars

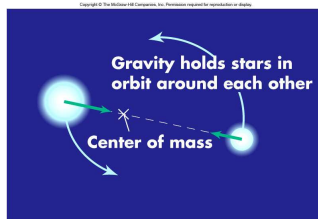
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## Binary Stars

- Two stars that revolve around each other as a result of their mutual gravitational attraction are called **binary stars**
- Binary star systems offer one of the few ways to measure stellar masses – and stellar mass plays the leading role in a star's evolution
- At least 40% of all stars known have orbiting companions (some more than one)
- Most binary stars are only a few AU apart – a few are even close enough to touch

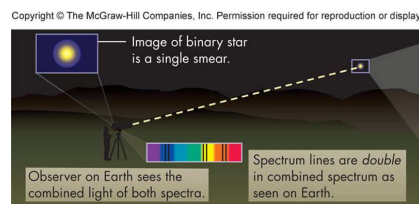
## Visual Binary Stars

- Visual binaries** are binary systems where we can directly see the orbital motion of the stars about each other by comparing images made several years apart



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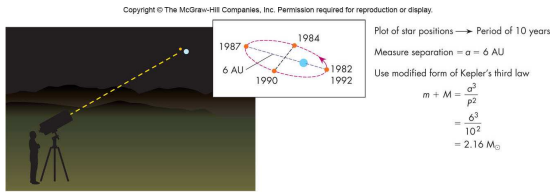
## Spectroscopic Binaries



- Spectroscopic binaries** are systems that are inferred to be binary by a comparison of the system's spectra over time
- Doppler analysis of the spectra can give a star's speed and by observing a full cycle of the motion the orbital period and distance can be determined

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## Stellar Masses



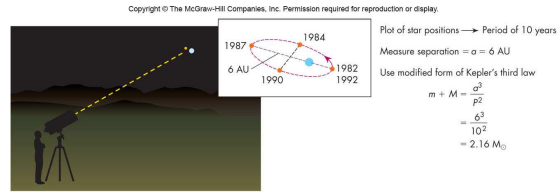
- Kepler's third law as modified by Newton is

$$(m + M)P^2 = a^3$$

- $m$  and  $M$  are the binary star masses (in solar masses),  $P$  is their period of revolution (in years), and  $a$  is the semimajor axis of one star's orbit about the other (in AU)

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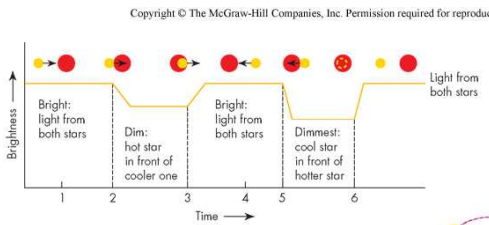
## Stellar Masses



- $P$  and  $a$  are determined from observations (may take a few years) and the above equation gives the combined mass ( $m + M$ )
- Further observations of the stars' orbit will allow the determination of each star's individual mass
- Most stars have masses that fall in the narrow range 0.1 to 30  $M_{\odot}$

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## Eclipsing Binaries

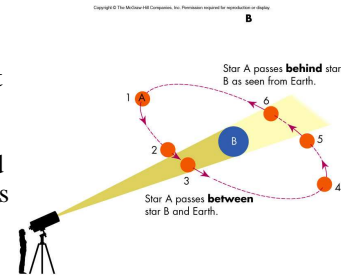


- A binary star system in which one star can eclipse the other star is called an **eclipsing binary**
- Watching such a system over time will reveal a combined light output that will periodically dim

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## Eclipsing Binaries

- The duration and manner in which the combined light curve changes together with the stars' orbital speed allows astronomers to determine the radii of the two eclipsing stars



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## Summary of Stellar Properties

- Distance
  - Parallax (triangulation) for nearby stars (distances less than 250 pc)
  - Standard-candle method for more distant stars
- Temperature
  - Wien's law (color-temperature relation)
  - Spectral class (O hot; M cool)
- Luminosity
  - Measure star's apparent brightness and distance and then calculate with inverse square law
  - Luminosity class of spectrum (to be discussed)
- Composition
  - Spectral lines observed in a star

Temperature (Kelvin)

## Summary of Stellar Properties

- Radius
  - Stefan-Boltzmann law (measure  $L$  and  $T$ , solve for  $R$ )
  - Interferometer (gives angular size of star; from distance and angular size, calculate radius)
  - Eclipsing binary light curve (duration of eclipse phases)
- Mass
  - Modified form of Kepler's third law applied to binary stars
- Radial Velocity
  - Doppler shift of spectrum lines

Temperature (Kelvin)

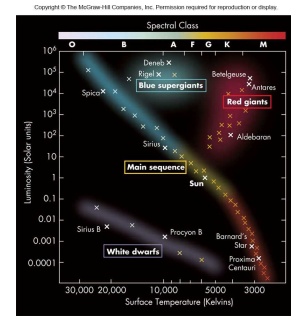


## Putting it all together – The Hertzsprung-Russell Diagram

- So far, only properties of stars have been discussed – this follows the historical development of studying stars
- The next step is to understand why stars have these properties in the combinations observed
- This step in our understanding comes from the H-R diagram, developed independently by Ejnar Hertzsprung and Henry Norris Russell in 1912

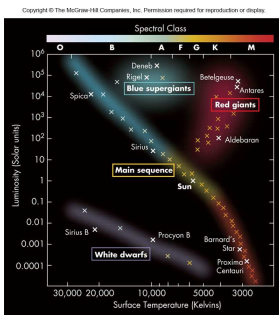
## The HR Diagram

- The *H-R diagram* is a plot of stellar temperature vs luminosity
- Interestingly, most of the stars on the H-R diagram lie along a smooth diagonal running from hot, luminous stars (upper left part of diagram) to cool, dim ones (lower right part of diagram)



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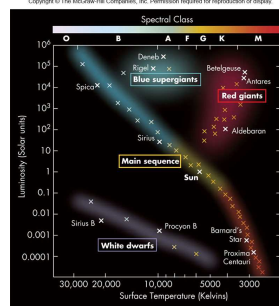
## The HR Diagram



- By tradition, bright stars are placed at the top of the H-R diagram and dim ones at the bottom, while high-temperature (blue) stars are on the left with cool (red) stars on the right (**Note:** temperature does not run in a traditional direction)

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## The HR Diagram



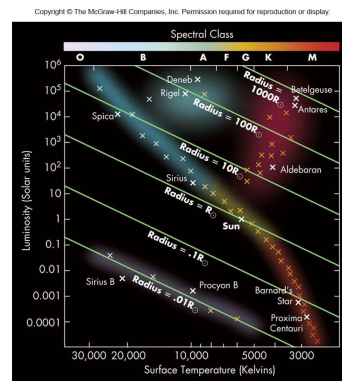
- The diagonally running group of stars on the H-R diagram is referred to as the *main sequence*
- Generally, 90% of a group of stars will be on the main sequence; however, a few stars will be cool but very luminous (upper right part of H-R diagram), while others will be hot and dim (lower left part of H-R diagram)

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## Analyzing the HR Diagram

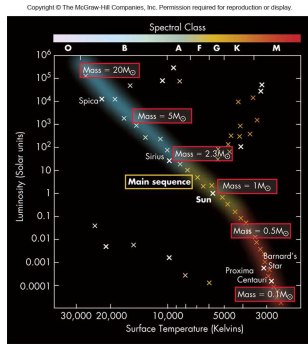
- The Stefan-Boltzmann law is a key to understanding the H-R diagram
  - For stars of a given temperature, the larger the radius, the larger the luminosity
  - Therefore, as one moves up the H-R diagram, a star's radius must become bigger
  - On the other hand, for a given luminosity, the larger the radius, the smaller the temperature
  - Therefore, as one moves right on the H-R diagram, a star's radius must increase
  - The net effect of this is that the smallest stars must be in the lower left corner of the diagram and the largest stars in the upper right

## Analyzing the HR Diagram



## Giants and Dwarfs

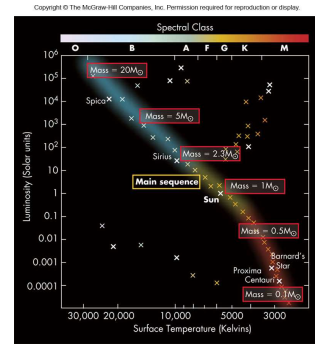
- Stars in the upper left are called **red giants** (red because of the low temperatures there)
- Stars in the lower right are **white dwarfs**
- Three stellar types: main sequence, red giants, and white dwarfs



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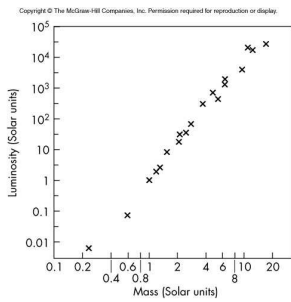
## Giants and Dwarfs

- Giants, dwarfs, and main sequence stars also differ in average density, not just diameter
- Typical density of main-sequence star is  $1 \text{ g/cm}^3$ , while for a giant it is  $10^{-6} \text{ g/cm}^3$



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## The Mass-Luminosity Relation



- Main-sequence stars obey a **mass-luminosity relation**, approximately given by:

$$L = M^3$$

- $L$  and  $M$  are measured in solar units
- Consequence: Stars at top of main-sequence are more massive than stars lower down

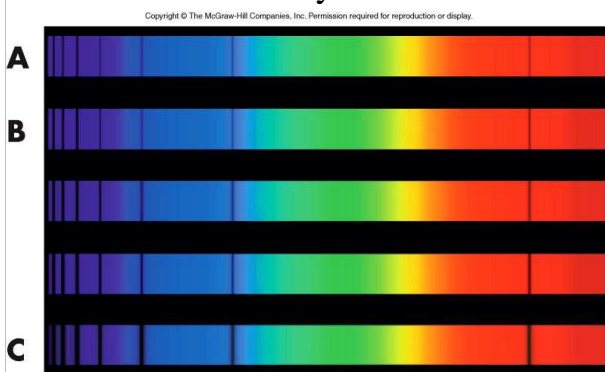
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## Luminosity Classes

- Another method was discovered to measure the luminosity of a star (other than using a star's apparent magnitude and the inverse square law)
  - It was noticed that some stars had very narrow absorption lines compared to other stars of the same temperature
  - It was also noticed that luminous stars had narrower lines than less luminous stars
- Width of absorption line depends on density: wide for high density, narrow for low density

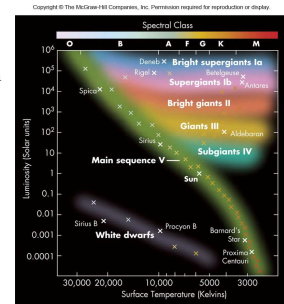
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## Luminosity Classes



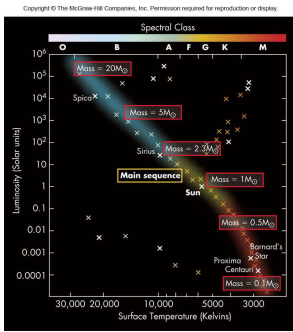
## Luminosity Classes

- Luminous stars (in upper right of H-R diagram) tend to be less dense, hence narrow absorption lines
- H-R diagram broken into luminosity classes: Ia (bright supergiant), Ib (supergiants), II (bright giants), III (giants), IV (subgiants), V (main sequence)
  - Star classification example: The Sun is G2V



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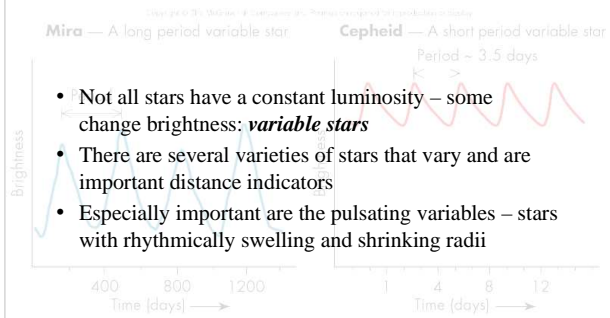
## Summary of the HR Diagram



- Most stars lie on the main sequence
  - Of these, the hottest stars are blue and more luminous, while the coolest stars are red and dim
  - Star's position on sequence determines its mass, being more near the top of the sequence
- Three classes of stars:
  - Main-sequence
  - Giants
  - White dwarfs

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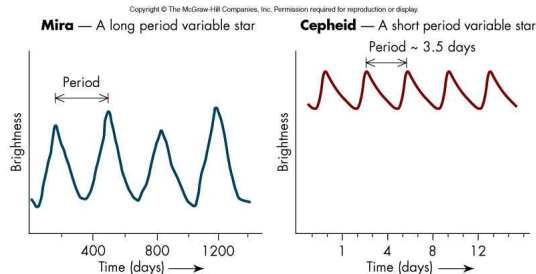
## Variable Stars



- Not all stars have a constant luminosity – some change brightness; **variable stars**
- There are several varieties of stars that vary and are important distance indicators
- Especially important are the pulsating variables – stars with rhythmically swelling and shrinking radii

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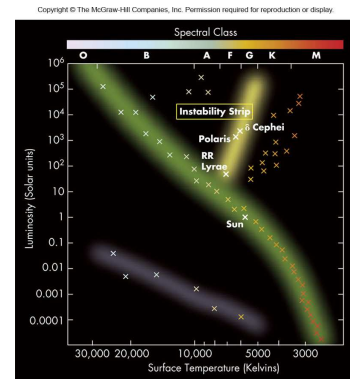
## Mira and Cepheid Variables



- Variable stars are classified by the shape and period of their light curves – Mira and Cepheid variables are two examples

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## The Instability Strip



- Most variable stars plotted on H-R diagram lie in the narrow “instability strip”

## Method of Standard Candles

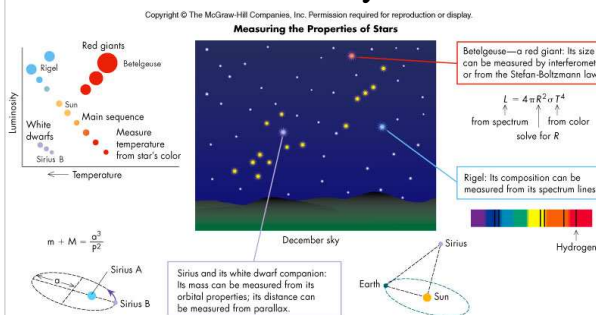
- Step 1: Measure a star's brightness (B) with a photometer
- Step 2: Determine star's Luminosity, L
- Use combined formula to calculate d, the distance to the star
- Sometimes easier to use ratios of distances
  - Write Inverse-Square Law for each star
  - Take the ratio:

$$B_{near} = \frac{L_{near}}{4\pi d_{near}^2}, B_{far} = \frac{L_{far}}{4\pi d_{far}^2}$$

$$\frac{B_{near}}{B_{far}} = \left( \frac{d_{far}}{d_{near}} \right)^2$$

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## Summary



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