Axisymmetric Gravitational Collapse

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- Plan of talk:
  - ◇ Goals/Motivation
  - ◇ Method
  - ◇ Equations
  - ◇ Results
  - ◇ Tests
  - ◇ Conclusion

Why study axisymmetry?

- Study Critical Phenomena
  - ◇ Less symmetry
  - ◇ GR and EM collapse

- Bridge the $1D \to 3D$ gap
  - ◇ easier than 3d
  - ◇ more interesting than 1D
  - ◇ test 3d results
  - ◇ develop techniques for 3D (*e.g.* BC's)

- Black Hole Excision and merger

- Hoop Conjecture, Cosmic Censorship
  - ◇ repeat `Shapiro & Teukolsky (PRL, 66 1991)`

Why is axisymmetry hard?

- Historically, difficult to handle axis/origin
- Boundary conditions and Stability
- Constraint equations need solving quickly
- Hard to get resolution

Method

- Cylindrical coordinates (ρ, z, φ)
  - $\diamond$ enforce smoothness on axis $\rho = 0$

- Spatial Metric:

$$ds^2 = \psi^4 \left(dz^2 + d\rho^2 + e^{2\rho\bar{\sigma}} \rho^2 d\varphi^2 \right)$$

  - $\diamond$ metric functions: $\psi(t, \rho, z)$, $\bar{\sigma}(t, \rho, z)$
  - $\diamond$ shift: $\beta^\rho(t, \rho, z)$, $\beta^z(t, \rho, z)$
  - $\diamond$ lapse: $\alpha(t, \rho, z)$

- Iterative Crank-Nicholson
  - $\diamond$ Implemented with RNPL

- Constrained Evolution:
  - $\diamond$ Fully/Partially constrained Multigrid solver
  - $\diamond$ Bi-Conjugate Gradient Solver
  - $\diamond$ Line/Pure relaxation solver
  - $\diamond$ Direct solver — invert the Jacobian

Equations

- (2+1)+1 Decomposition
  - ◇ developed by Geroch
  - ◇ used by Nakamura in the 1980s
  - ◇ divide out symmetry about φ

- Maximal Slicing: $K \equiv 0$
  - ◇ Use $\bar{\sigma}$ and its "conjugate" $\bar{\Omega}$ where

$$K_\varphi{}^\varphi = \frac{1}{3\alpha}\left(2\alpha\rho\bar{\Omega} + \beta^\rho{}_{,\rho} - \beta^z{}_{,z}\right)$$

- Metric Component Equations:
  - ◇ Elliptic Constraints: $\psi, \alpha, \beta^\rho, \beta^z$
  - ◇ Evolution Equations: ψ, $\bar{\sigma}$, $\bar{\Omega}$
  - ◇ Choice: evolution or constraint for ψ

- Matter Component Equations:
  - ◇ **Brill/GR wave:** vacuum!
  - ◇ **Scalar Field** ϕ: evolution
  - ◇ **E&M** A^μ: evolution/constraint

<u>**Empty**</u>

Initial Data

- Vacuum:

$$\begin{aligned}
\bar{\sigma}(0,\rho,z) &= A\rho e^{-\left(\frac{\sqrt{(\rho-\rho_0)^2+\epsilon(z-z_0)^2}-R_0}{\Delta}\right)^2} \\
\bar{\Omega}(0,\rho,z) &= 0 \\
\phi(0,\rho,z) &= 0 \\
\phi_t(0,\rho,z) &= 0
\end{aligned}$$

Generalized Brill Waves

- Scalar:

$$\begin{aligned}
\bar{\sigma}(0,\rho,z) &= 0 \\
\bar{\Omega}(0,\rho,z) &= 0 \\
\phi(0,\rho,z) &= \text{arbitrary} \\
\phi_t(0,\rho,z) &= \text{arbitrary}
\end{aligned}$$

- Combinations of above

- NB: E&M a bit more complicated

Boundary Conditions

- On axis:
  - ◇ α, ψ, β^ρ, β^z — even
  - ◇ $\bar{\sigma}$, $\bar{\Omega}$ — odd
- Outer boundaries:
  - ◇ β^ρ, β^z — fall off as $1/r$; dirichlet
  - ◇ α, ψ — fall off as $1 + C/r$; dirichlet
  - ◇ $\bar{\sigma}$, $\bar{\Omega}$ — OWBC; any other ideas?

Mass estimates

- ADM mass, M_{ADM}:

$$M_{\mathrm{ADM}} = \frac{1}{16\pi} \lim_{r\to\infty} \int \left({h^a}_{b;a} - {h^a}_{a;b} \right) N^b dA$$

- Integrated mass density:

$$\begin{aligned} M &= -\frac{1}{2\pi} \int \nabla^2 \psi \; dV \\ &= -\frac{1}{3} \int d\rho \; dz \left(\left[\rho e^\sigma \psi^3_{,\rho} \right]_{,\rho} + \left[\rho e^\sigma \psi^3_{,z} \right]_{,z} \right) \end{aligned}$$

Results

- Stable and convergent!

- Duplicates Spherical Symmetry Code

- Excision allows long time evol. of BHs

- Sample running times:

1 proc Origin 200 for 5 crossing times

512×1024	24 days (expected)	336 MB
256×512	58 hours	84 MB
128×256	6.5 hours	21 MB
64×128	40 minutes	5.3 MB

Tests...

- Re-derive and Compare Equations using Maple

- Compare to Spherical Symmetry code
  - Converges to spherically symmetric results

- Second-order Convergence

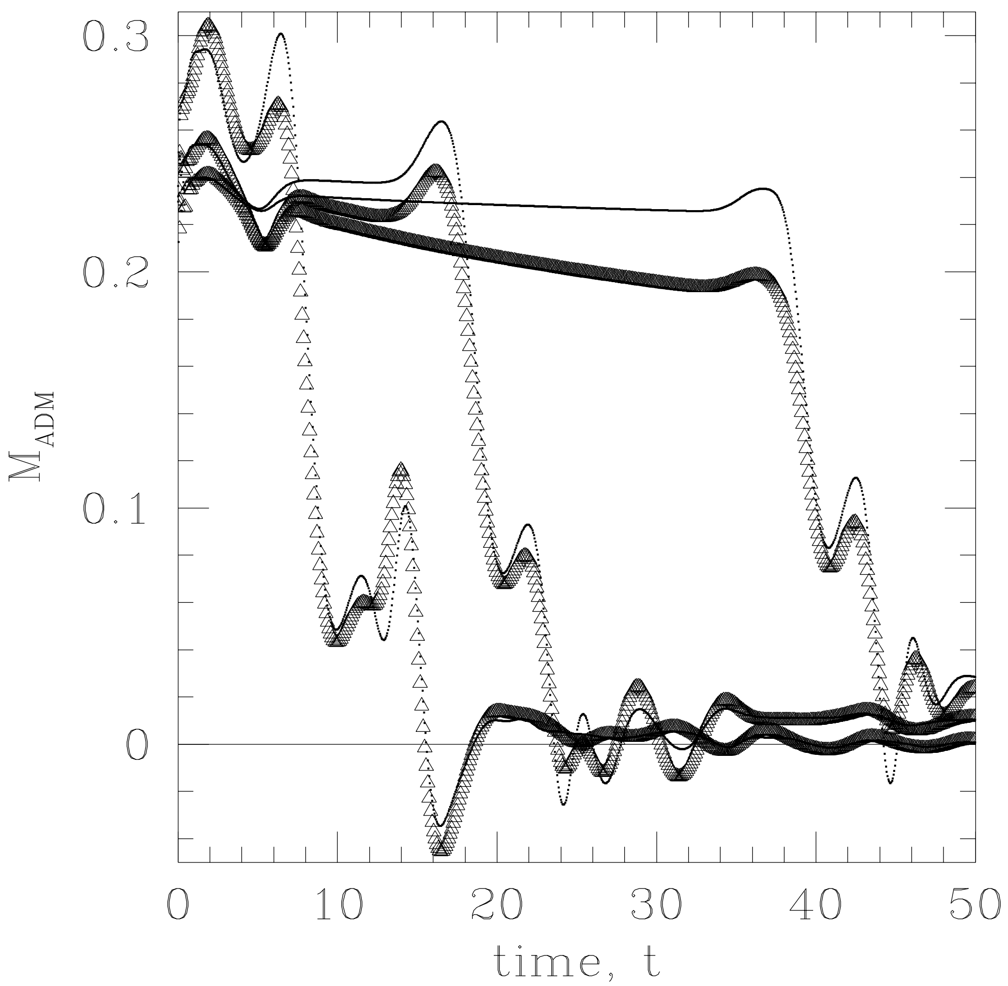
- Ind. Residuals converge to zero

- Extensive tests of constraint solver(s)

- In future: compare to other codes

Tests...

- Conservation of Mass estimates
  - ◇ General initial data:



Conclusion...Future

- It works:
  - ◇ keep testing and start writing
  - ◇ keep running and exploring

- Problems:
  - ◇ robustness of elliptic solver for Brill
  - ◇ need more resolution

- Still to do:
  - ◇ resolution (AMR)
  - ◇ better Boundary Conditions
  - ◇ test E&M
  - ◇ add rotation (straight-forward w/ (2+1)+1)
  - ◇ apparent horizon finder
  - ◇ speed (distributed via MPI)